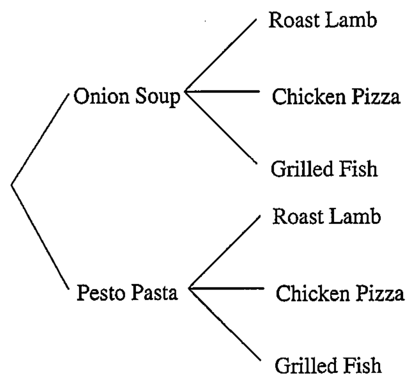
5 Probability

Probability is concerned with the chance or likelihood of a particular identified outcome occurring. The probability, p, of a particular event is given by:

$$p = \frac{\text{number of ways in which that event can occur}}{\text{total number of elements in sample space}}$$

The *sample space* is the set of all possible outcomes. Consider dinner options at a fancy restaurant. There are two choices for entree: onion soup or pesto pasta, and three choices for main: roast lamb, chicken pizza, or grilled fish. What is the probability that if you chose a meal of one entree and one main at random, you would get onion soup and chicken pizza? First let's list the sample space. It can be helpful to use a table or tree diagram to list sample spaces.



Here we can see that there are 6 meal options. Therefore the probability of choosing a meal of onion soup and chicken pizza is $\frac{1}{6}$.

Practice Question 1

You now have the choice of 3 entrees: onion soup, pesto pasta, mini sausage; 4 mains: roast lamb, chicken pizza, grilled fish, beef stir-fry; and 5 desserts: strawberry sundae, chocolate ice-cream, caramel slice, apple pie or fruit salad. What is the size of the sample space, ie. how many meal combinations are there? Also what is the probability that a meal chosen at random contains grilled fish as the main?

Sometimes we are concerned with a probability that involves two events that can happen together. For example, in a deck of cards I might need a heart or a 7 in order to get a winning hand. We can't just add the probability of getting a 7 to the probability of getting a heart, because then we have counted the 7 of hearts twice. In reality there is only one 7 of hearts. The principle of inclusion/exclusion says that if A and B are events then

$$\text{Prob}(A \cup B) = \text{Prob}(A) + \text{Prob}(B) - \text{Prob}(A \cap B)$$

This means, the probability of A **or** B happening is equal to the probability of A plus the probability of B *minus* the probability of A **and** B happening at the same time. So the probability of getting a heart or a 7 is

$$P(7 \cup H) = P(7) + P(H) - P(7 \cap H) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$$

Practice Question 2

Looking back at the meal question where there were 3 entrees, 4 mains and 5 desserts, what is the probability that a meal chosen at random contains onion soup or a strawberry sundae?

There are times when we might know something about the event that is going to occur; this can reduce the sample size. For example, what is the probability of drawing a 7 from a pack of cards if I know that the card drawn is a number card (2,3,...,10)? We no longer have to consider the possibility of drawing a picture card. This means we are now looking a smaller sample space. When we have extra information we use *conditional probability*, which is the probability that A occurs **given that** B occurs. We write this as $\text{Prob}(A|B)$. The formula for calculating this is:

$$\text{Prob}(A|B) = \frac{\text{Prob}(A \cap B)}{\text{Prob}(B)}$$

In our example with the cards, the probability of drawing a 7 given that it is a number card is then:

$$\frac{4}{52} \div \frac{36}{52} = \frac{1}{9}$$

Practice Question 3

What is the probability of getting an even number when you toss a die given that the value is greater than 3?

Discussion Questions

Work through these problems with the person next to you or in a small group.

1. There are two identical baskets. One contains two blue balls and a yellow ball; the other contains two yellow balls. If a basket is chosen at random and a ball selected at random from that basket, what is the probability of selecting a yellow ball?

2. Out of forty students, 14 are taking English and 29 are taking Biology. There are five students studying both. A student is chosen at random.
   (a) What's the probability the student is studying either English or Biology?
   (b) What's the probability the student is studying neither?
   (c) What's the probability the student is studying only Biology?

3. A survey of a class of 24 students was completed on the flavour of ice-cream they ate last night. There were 6 students who ate chocolate, 12 students who ate strawberry and 15 students who ate vanilla. There was only 1 who ate all three flavours, 2 who ate chocolate and vanilla and 2 who ate chocolate and strawberry. Everyone ate at least one flavour and 16 students ate only one flavour. If a student was picked at random from the group, what's the probability that they ate both vanilla and strawberry ice-cream last night?

4. Mary is getting married and needs to organise a seating plan. There are 6 people in the bridal party but only 3 chairs to the right of the bride. How many different seating arrangements are there for those 3 chairs?