1. (a) $f(x) = 2\sqrt{x^2}$

When determining the domain of this function, we need to keep in mind the following:

* we can only take the square root of positive numbers or 0, so $x^2 \geq 0$;
* we can square any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of x can be substituted into f .

(b) $f(w) = \dfrac{-3}{w^2+3}$

When determining the domain of this function, we need to keep in mind the following:

* denominator of a fraction cannot be 0, so $w^2 + 3 \neq 0$;
* so $w^2 \neq -3$;
* we can square any number and result will always be a positive number or 0.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of w can be substituted into f .

(c) $f(z) = \sqrt{3\,|z|}$

When determining the domain of this function, we need to keep in mind the following:

* we can only take the square root of positive numbers or 0, so $3\,|z| \geq 0$;
* we can find the absolute value of any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of z can be substituted into f .

When evaluating the range, we need to keep in mind the following (starting with variable z):

* absolute value is always positive or 0, so $|z| \geq 0$;
* square root is always positive or 0, so $\sqrt{3\,|z|} \geq 0$.

Hence, the range of this function is $[0, \infty)$.

(d) $f(x) = \left|-8 \times \dfrac{-10}{x}\right|$

When evaluating the range, we need to keep in mind the following (starting with variable x):

* numerator is not 0, fraction can't be 0, so $\dfrac{-10}{x} \neq 0$;
* multiplying by a negative number usually reverses the inequality , so $-8 \times \dfrac{-10}{x} \neq 0$;
* absolute value is always positive or 0, so $\left|-8 \times \dfrac{-10}{x}\right| > 0$.

Hence, the range of this function is $(0, \infty)$.

(e) $f(z) = \dfrac{4}{\sqrt{z}+10}$

When evaluating the range, we need to keep in mind the following (starting with variable z):

* square root is always positive or 0, so $0 \leq \sqrt{z}$;
* fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
* so $10 \leq \sqrt{z} + 10$.

Hence, the range of this function is $(0, \frac{2}{5}]$.

2. To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n.

(a) $8z\,(4z+7) = 0$, so

$$8z = 0 \qquad \textit{or} \qquad 4z + 7 = 0$$
$$z = 0 \qquad\qquad 4z = -7$$
$$z = -\frac{7}{4}$$

(b) $(-5-6y)\,(3y+1) = 0$, so

$$-5 - 6y = 0 \qquad \textit{or} \qquad 3y + 1 = 0$$
$$-6y = 5 \qquad\qquad 3y = -1$$
$$y = -\frac{5}{6} \qquad\qquad y = -\frac{1}{3}$$

(c) $(y-3)\,(7+3y) = 0$, so

$$y - 3 = 0 \qquad \textit{or} \qquad 7 + 3y = 0$$
$$y = 3 \qquad\qquad 3y = -7$$
$$y = -\frac{7}{3}$$

(d) $(3+10x)^2 = 0$, so $3 + 10x = 0$, so $10x = -3$, so $x = -\frac{3}{10}$

3. $-2x^2 + 3x - 12 = 0$, so we use $a = -2, b = 3, c = -12$ in the quadratic formula. Hence

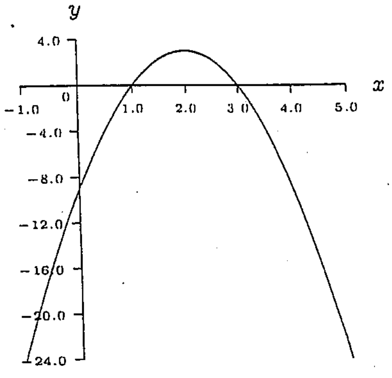
$$\begin{aligned} x &= \frac{-3 \pm \sqrt{3^2 - 4 \times (-2) \times (-12)}}{2 \times (-2)} \\ &= \frac{-3 \pm \sqrt{9 - 96}}{-4} \\ &= \frac{-3 \pm \sqrt{-87}}{-4} \end{aligned}$$

Hence there is no solution.

4. (a) The roots of $y = -3x^2 + 12x - 9$ are the x values that satisfy $-3x^2 + 12x - 9 = 0$. You can solve this equation either by using the quadratic formula or by factoring. Here we will use factoring.

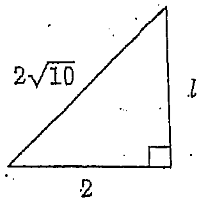
First divide through by -3 to get $x^2 - 4x + 3 = 0$. Now because $x^2 - 4x + 3 = (x-1)\,(x-3)$, the two roots of the quadratic equation are $x = 1$, 3.

(b) The y-intercept occurs when $x = 0$, so substituting this into $y = -3x^2 + 12x - 9$ gives $y = -9$.



(c)

5. (a) $(2\sqrt{10})^2 = 2^2 + l^2 \Rightarrow 40 = 4 + l^2 \Rightarrow l^2 = 36 \Rightarrow l = 6$. Window is 7m high, so he cannot reach.



(b) $y = 2x + c$, $(0,0)$ is on the line $\Rightarrow$ $0 = 2 \times 0 + c \Rightarrow$ $c = 0 \Rightarrow$ $y = 2x$

(c) We know the equation of the ladder is $y = 2x$. When $x = 2\sqrt{2}$, $y = 2x = 2 \times 2\sqrt{2} = 4\sqrt{2} \Rightarrow$ window is $4\sqrt{2}$m high.

(d) When they have travelled half of the way down the ladder, their point must have an x-coordinate of $\sqrt{2}$. Hence the equation of the vertical line through which they fall is $x = \sqrt{2}$