MATH1040/7040 **Assignment 6 Solutions**

1. 1. $y = e^{-6x}$, which is a graph of exponential decay. Hence the matching graph is Graph A.

2. $6y - 9x = 8y - 9x - 8$, so $-2y = -8$, so $y = 4$. Hence this is a horizontal line, with y positive. Hence the matching graph is Graph N.

3. $11 = 6x + 13$, so $6x = -2$, so $x = -\frac{2}{6}$. Hence this is a vertical line, with x negative. Hence the matching graph is Graph H.

4. $-x^2 - 10 = 7y - 7x^2 - 16$, so $7y = 6x^2 + 6$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is positive. Hence the matching graph is Graph B.

5. $y = 5 \times |-3x|$, so $y = 5 \times |3x|$, which is a graph of absolute value. Hence the matching graph is Graph G.

6. $10x + 12 = -13y + 12$, so $13y = -10x$. Hence this is a straight line, with negative gradient and passing through the origin. Hence the matching graph is Graph I.

7. $y = -14 \times |-6x|$, so $y = -14 \times |6x|$, which is a graph of negative absolute value. Hence the matching graph is Graph E.

8. $-7y - 7x + 2 = 14y + 15x$, so $21y = -22x + 2$. Hence this is a straight line, with negative gradient and positive y-intercept. Hence the matching graph is Graph M.

2. Let P be the amount invested, r be the interest rate per time period, n be the number of time periods and F be the final value. In each case, $P = 100$. Then:

1. Interest compounds annually, so we use the rate and number of time periods given in the question.
   Hence $r = 6.0\% = 0.06$ and $n = 1$, so $F = 100 \times (1 + 0.06)^1 = 100 \times 1.06^1 = 106.00$.
   The final balance is \$106.00.
2. Interest compounds twice a year, so we need to halve the rate and double the number of time periods given in the question.
   Hence $r = 3.0\% = 0.03$ and $n = 2$, so $F = 100 \times (1 + 0.03)^2 = 100 \times 1.03^2 \approx 106.09$.
   The final balance is \$106.09.
3. Interest compounds 4 times a year, so we need to divide the given rate by 4 and multiply the given number of years by 4.
   Hence $r = 1.5\% = 0.015$ and $n = 4$, so $F = 100 \times (1 + 0.015)^4 = 100 \times 1.015^4 \approx 106.14$.
   The final balance is \$106.14.
4. Interest compounds 12 times a year, so we need to divide the given rate by 12 and multiply the given number of years by 12.
   Hence $r = 0.5\% = 0.005$ and $n = 12$, so $F = 100 \times (1 + 0.005)^{12} = 100 \times 1.005^{12} \approx 106.17$.
   The final balance is \$106.17.
5. Interest compounds continuously, so $F = 100e^{0.06 \times 1} = 100e^{0.06} \approx 106.18$.
   The final balance is \$106.18.

(continued over...)

3. Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \dfrac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 1000e^{-0.07\times 6} \\ &= 1000e^{-0.42} \\ &\approx 657.05 \end{aligned}$$

Hence he needs to invest approximately \$657.05 .

4. Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding monthly . Then

$r = 1 \times \dfrac{10.0}{12} = 0.83$ percent $= 0.0083$, and

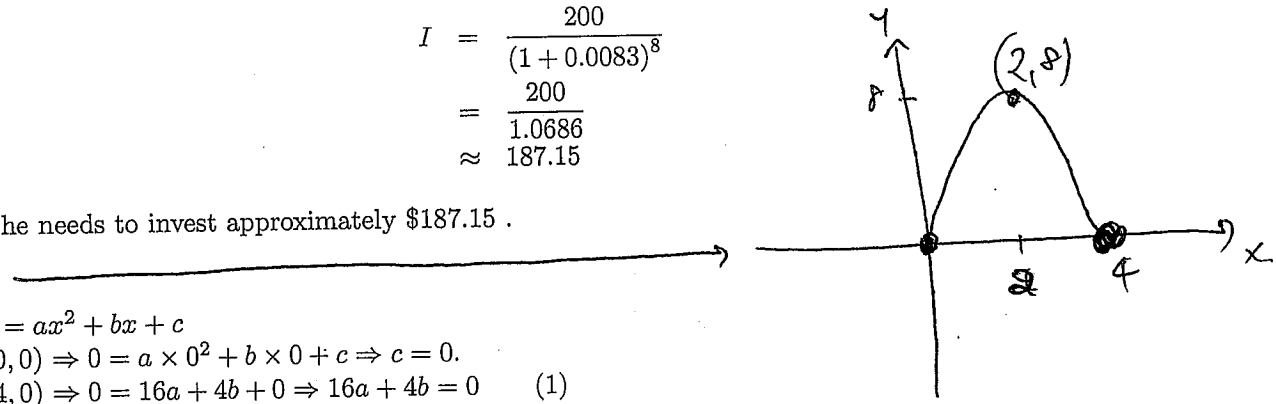
$n = 8 \div 1 = 8$

$B = I(1+r)^n$, so $I = \dfrac{B}{(1+r)^n}$. Therefore

$$\begin{aligned} I &= \frac{200}{(1+0.0083)^8} \\ &= \frac{200}{1.0686} \\ &\approx 187.15 \end{aligned}$$

Hence he needs to invest approximately \$187.15 .

5. (a)



(b) $y = ax^2 + bx + c$

$(0,0) \Rightarrow 0 = a \times 0^2 + b \times 0 + c \Rightarrow c = 0.$

$(4,0) \Rightarrow 0 = 16a + 4b + 0 \Rightarrow 16a + 4b = 0 \quad (1)$

$(2,8) \Rightarrow 8 = 4a + 2b + 0 \Rightarrow 4a + 2b = 8 \quad (2)$

(c) Solve (1) and (2) simultaneously: $16a + 4b = 0$ (1) $4a + 2b = 8$ (2)
From (2) $\Rightarrow 2b = 8 - 4a \Rightarrow b = 4 - 2a$, substitute into (1) $\Rightarrow 16a + 4(4 - 2a) = 0 \Rightarrow 16a + 16 - 8a = 0 \Rightarrow 8a + 16 = 0 \Rightarrow 8a = -16 \Rightarrow a = -2$ and substitute into (2) $\Rightarrow 4 \times -2 + 2b = 8 \Rightarrow -8 + 2b = 8 \Rightarrow 2b = 16 \Rightarrow b = 8$.
Hence the equation is $y = -2x^2 + 8x$.

(d) $y = -4x + c$. Now $(4.5, 0)$ is on the line, so $0 = -4 \times 4.5 + c \Rightarrow c = 18$ and $y = -4x + 18$.

(e) $y = -4x + 18$ (1) and $y = -2x^2 + 8x$ (2)
We solve (1) and (2) simultaneously by equating y-values, so
$-4x + 18 = -2x^2 + 8x \Rightarrow -2x^2 + 12x - 18 = 0 \Rightarrow$

$$x = \frac{-12 \pm \sqrt{144 - 4 \times -2 \times -18}}{-4} = \frac{-12 \pm \sqrt{0}}{-4} = 3.$$

When $x = 3$, $y = -4 \times 3 + 18 = 6 \Rightarrow$ point is $(3, 6)$.

6. BONUS QUESTION 2 marks (Only 1 mark if correct answer but no working.)

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}} = 1 + \cfrac{1}{1 + \cfrac{1}{\frac{3}{2}}} = 1 + \cfrac{1}{1 + \frac{2}{3}} = 1 + \cfrac{1}{\frac{5}{3}}$$

$$= 1 + \frac{3}{5} = 1\frac{3}{5}.$$