

1. (1) Let P be the final population in millions. Then

$$\begin{aligned} P &= 400e^{0.08 \times 7} \\ &= 400e^{0.56} \\ &\approx 700.27 \end{aligned}$$

Hence the final population is approximately 700.27 million bacteria.

- (2) Let A be the final amount of the material remaining. Then

$$\begin{aligned} A &= 100e^{-0.05 \times 20} \\ &= 100e^{-1} \\ &\approx 36.79 \end{aligned}$$

Hence the amount of material remaining after 20 thousand years is approximately 36.79 units.

- (3) Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \frac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 900e^{-0.04 \times 5} \\ &= 900e^{-0.2} \\ &\approx 736.86 \end{aligned}$$

Hence he needs to invest approximately \$736.86 .

- (4) Let F be the final account balance. Then

$$\begin{aligned} F &= 400e^{0.05 \times 12} \\ &= 400e^{0.6} \\ &\approx 728.85 \end{aligned}$$

Hence the final account balance is approximately \$728.85 .

- (5) Let F be the final amount he needs, I be the amount he has to invest, r be the interest rate and t be the number of years. Then $F = Ie^{rt}$ so $e^{rt} = \frac{F}{I}$, so $rt = \ln \frac{F}{I}$, and $t = \left(\ln \frac{F}{I}\right) \div r$. Then

$$\begin{aligned} t &= \left(\ln \frac{1710}{900}\right) \div 0.01 \\ &= (\ln 1.90) \div 0.01 \\ &\approx 0.64 \div 0.01 \\ &\approx 64.19 \end{aligned}$$

Hence he needs to invest \$900 for approximately 64.19 years. Therefore Damien can marry Celeste when he is about 79 years old.

- (6) Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding monthly . Then

$$r = 1 \times \frac{8.0}{12} = 0.67 \text{ percent} = 0.0067, \text{ and}$$

$$n = 5 \div 1 = 5$$

$$B = I(1+r)^n, \text{ so } I = \frac{B}{(1+r)^n}. \text{ Therefore}$$

$$\begin{aligned} I &= \frac{100}{(1+0.0067)^5} \\ &= \frac{100}{1.0338} \\ &\approx 96.73 \end{aligned}$$

Hence he needs to invest approximately \$96.73 .

2. (1) Let P be the final population in millions. Then

$$\begin{aligned} P &= 600e^{0.07 \times 9} \\ &= 600e^{0.63} \\ &\approx 1126.57 \end{aligned}$$

Hence the final population is approximately 1126.57 million bacteria.

- (2) Let A be the final amount of the material remaining. Then

$$\begin{aligned} A &= 600e^{-0.04 \times 13} \\ &= 600e^{-0.52} \\ &\approx 356.71 \end{aligned}$$

Hence the amount of material remaining after 13 thousand years is approximately 356.71 units.

- (3) Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \frac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 500e^{-0.03 \times 7} \\ &= 500e^{-0.21} \\ &\approx 405.29 \end{aligned}$$

Hence he needs to invest approximately \$405.29 .

- (4) Let F be the final account balance. Then

$$\begin{aligned} F &= 100e^{0.02 \times 17} \\ &= 100e^{0.34} \\ &\approx 140.49 \end{aligned}$$

Hence the final account balance is approximately \$140.49 .

- (5) Let F be the final amount he needs, I be the amount he has to invest, r be the interest rate and t be the number of years. Then $F = Ie^{rt}$ so $e^{rt} = \frac{F}{I}$, so $rt = \ln \frac{F}{I}$, and $t = \left(\ln \frac{F}{I}\right) \div r$. Then

$$\begin{aligned} t &= \left(\ln \frac{2160}{800}\right) \div 0.06 \\ &= (\ln 2.70) \div 0.06 \\ &\approx 0.99 \div 0.06 \\ &\approx 16.55 \end{aligned}$$

Hence he needs to invest \$800 for approximately 16.55 years. Therefore Damien can marry Celeste when he is about 30 years old.

- (6) Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding monthly . Then

$$r = 1 \times \frac{3.0}{12} = 0.25 \text{ percent} = 0.0025, \text{ and}$$

$$n = 23 \div 1 = 23$$

$$B = I(1+r)^n, \text{ so } I = \frac{B}{(1+r)^n}. \text{ Therefore}$$

$$\begin{aligned} I &= \frac{400}{(1+0.0025)^{23}} \\ &= \frac{400}{1.0591} \\ &\approx 377.68 \end{aligned}$$

Hence he needs to invest approximately \$377.68 .

3. (1) Let P be the final population in millions. Then

$$\begin{aligned} P &= 900e^{0.04 \times 11} \\ &= 900e^{0.44} \\ &\approx 1397.44 \end{aligned}$$

Hence the final population is approximately 1397.44 million bacteria.

- (2) Let A be the final amount of the material remaining. Then

$$\begin{aligned} A &= 800e^{-0.09 \times 9} \\ &= 800e^{-0.81} \\ &\approx 355.89 \end{aligned}$$

Hence the amount of material remaining after 9 thousand years is approximately 355.89 units.

- (3) Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \frac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 1000e^{-0.06 \times 13} \\ &= 1000e^{-0.78} \\ &\approx 458.41 \end{aligned}$$

Hence he needs to invest approximately \$458.41 .

- (4) Let F be the final account balance. Then

$$\begin{aligned} F &= 200e^{0.01 \times 10} \\ &= 200e^{0.1} \\ &\approx 221.03 \end{aligned}$$

Hence the final account balance is approximately \$221.03 .

- (5) Let F be the final amount he needs, I be the amount he has to invest, r be the interest rate and t be the number of years. Then $F = Ie^{rt}$ so $e^{rt} = \frac{F}{I}$, so $rt = \ln \frac{F}{I}$, and $t = \left(\ln \frac{F}{I}\right) \div r$. Then

$$\begin{aligned} t &= \left(\ln \frac{720}{300}\right) \div 0.05 \\ &= (\ln 2.40) \div 0.05 \\ &\approx 0.88 \div 0.05 \\ &\approx 17.51 \end{aligned}$$

Hence he needs to invest \$300 for approximately 17.51 years. Therefore Damien can marry Celeste when he is about 35 years old.

- (6) Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding monthly . Then

$$r = 1 \times \frac{3.0}{12} = 0.25 \text{ percent} = 0.0025, \text{ and}$$

$$n = 10 \div 1 = 10$$

$$B = I(1+r)^n, \text{ so } I = \frac{B}{(1+r)^n}. \text{ Therefore}$$

$$\begin{aligned} I &= \frac{100}{(1+0.0025)^{10}} \\ &= \frac{100}{1.0253} \\ &\approx 97.53 \end{aligned}$$

Hence he needs to invest approximately \$97.53 .

4. (1) Let P be the final population in millions. Then

$$\begin{aligned} P &= 800e^{0.06 \times 14} \\ &= 800e^{0.84} \\ &\approx 1853.09 \end{aligned}$$

Hence the final population is approximately 1853.09 million bacteria.

- (2) Let A be the final amount of the material remaining. Then

$$\begin{aligned} A &= 1000e^{-0.04 \times 16} \\ &= 1000e^{-0.64} \\ &\approx 527.29 \end{aligned}$$

Hence the amount of material remaining after 16 thousand years is approximately 527.29 units.

- (3) Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \frac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 400e^{-0.07 \times 10} \\ &= 400e^{-0.7} \\ &\approx 198.63 \end{aligned}$$

Hence he needs to invest approximately \$198.63 .

- (4) Let F be the final account balance. Then

$$\begin{aligned} F &= 900e^{0.09 \times 14} \\ &= 900e^{1.26} \\ &\approx 3172.88 \end{aligned}$$

Hence the final account balance is approximately \$3172.88 .

- (5) Let F be the final amount he needs, I be the amount he has to invest, r be the interest rate and t be the number of years. Then $F = Ie^{rt}$ so $e^{rt} = \frac{F}{I}$, so $rt = \ln \frac{F}{I}$, and $t = \left(\ln \frac{F}{I}\right) \div r$. Then

$$\begin{aligned} t &= \left(\ln \frac{1190}{700}\right) \div 0.10 \\ &= (\ln 1.70) \div 0.10 \\ &\approx 0.53 \div 0.10 \\ &\approx 5.31 \end{aligned}$$

Hence he needs to invest \$700 for approximately 5.31 years. Therefore Damien can marry Celeste when he is about 24 years old.

- (6) Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding quarterly . Then

$$r = 3 \times \frac{9.0}{12} = 2.25 \text{ percent} = 0.0225, \text{ and}$$

$$n = 21 \div 3 = 7$$

$$B = I(1+r)^n, \text{ so } I = \frac{B}{(1+r)^n}. \text{ Therefore}$$

$$\begin{aligned} I &= \frac{300}{(1+0.0225)^7} \\ &= \frac{300}{1.1685} \\ &\approx 256.73 \end{aligned}$$

Hence he needs to invest approximately \$256.73 .

5. (1) Let P be the final population in millions. Then

$$\begin{aligned} P &= 1000e^{0.05 \times 16} \\ &= 1000e^{0.8} \\ &\approx 2225.54 \end{aligned}$$

Hence the final population is approximately 2225.54 million bacteria.

- (2) Let A be the final amount of the material remaining. Then

$$\begin{aligned} A &= 300e^{-0.05 \times 10} \\ &= 300e^{-0.5} \\ &\approx 181.96 \end{aligned}$$

Hence the amount of material remaining after 10 thousand years is approximately 181.96 units.

- (3) Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \frac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 200e^{-0.01 \times 3} \\ &= 200e^{-0.03} \\ &\approx 194.09 \end{aligned}$$

Hence he needs to invest approximately \$194.09 .

- (4) Let F be the final account balance. Then

$$\begin{aligned} F &= 900e^{0.09 \times 9} \\ &= 900e^{0.81} \\ &\approx 2023.12 \end{aligned}$$

Hence the final account balance is approximately \$2023.12 .

- (5) Let F be the final amount he needs, I be the amount he has to invest, r be the interest rate and t be the number of years. Then $F = Ie^{rt}$ so $e^{rt} = \frac{F}{I}$, so $rt = \ln \frac{F}{I}$, and $t = \left(\ln \frac{F}{I}\right) \div r$. Then

$$\begin{aligned} t &= \left(\ln \frac{920}{400}\right) \div 0.02 \\ &= (\ln 2.30) \div 0.02 \\ &\approx 0.83 \div 0.02 \\ &\approx 41.65 \end{aligned}$$

Hence he needs to invest \$400 for approximately 41.65 years. Therefore Damien can marry Celeste when he is about 52 years old.

- (6) Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding monthly . Then

$$r = 1 \times \frac{5.0}{12} = 0.42 \text{ percent} = 0.0042, \text{ and}$$

$$n = 20 \div 1 = 20$$

$$B = I(1+r)^n, \text{ so } I = \frac{B}{(1+r)^n}. \text{ Therefore}$$

$$\begin{aligned} I &= \frac{400}{(1+0.0042)^{20}} \\ &= \frac{400}{1.0867} \\ &\approx 368.08 \end{aligned}$$

Hence he needs to invest approximately \$368.08 .