MATH1040 Basic Mathematics — Practice Problems 5 SOLUTIONS

1. (**1**) Let $(x_1, y_1) = (-8, \sqrt{2})$ and $(x_2, y_2) = (-6, \sqrt{2})$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(-8-(-6))^2 + \left(\sqrt{2}-\sqrt{2}\right)^2} = \sqrt{(-2)^2 + 0^2} = \sqrt{4+0} = \sqrt{4}$.
Hence $d = 2$

(**2**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -y &= x + 2, \quad \text{so} \\ y &= -x - 2 \end{aligned}$$

Hence the gradient is $m = -1$ and the y-intercept is $c = -2$.

(**3**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -9x - 8 + 10y &= 6y - 3 + 8x, \text{ so} \\ 10y - 6y &= 8x + 9x - 3 + 8 \\ 4y &= 17x + 5 \\ y &= \frac{17}{4}x + \frac{5}{4} \end{aligned}$$

Hence the gradient is $m = \dfrac{17}{4}$ and the y-intercept is $c = \dfrac{5}{4}$.

(**4**) Thus the equation of the line is $y = 6x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (9, 10)$ into this equation to get the value for c. Hence $10 = 6 \times 9 + c$, so $-44 = c$.
Hence the equation of the line is $y = 6x - 44$.

(**5**) Let $(x_1, y_1) = (-8, 8)$ and $(x_2, y_2) = (1, 9)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.
Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9-8}{1-(-8)} = \dfrac{1}{9}$. Hence $m = \dfrac{1}{9}$.
Thus the equation of the line is $y = \dfrac{1}{9}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-8, 8)$ into this equation to get the value for c.
Hence $8 = \dfrac{1}{9} \times (-8) + c$, so $8 = -\dfrac{8}{9} + c$. Hence $c = 8 - \left(-\dfrac{8}{9}\right) = \dfrac{80}{9}$.
Hence the equation of the line is $y = \dfrac{1}{9}x + \dfrac{80}{9}$.

(**6**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -10y &= -60 - 30x, \text{ so} \\ y &= 3x + 6 \end{aligned}$$

Hence, the gradient of the original line is $m = 3$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = 3x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (3, 0)$ into this equation to get the value for c.
$0 = 3 \times 3 + c$, so $0 = 9 + c$. Hence $c = 0 - 9 = -9$.
Hence the equation of the line is $y = 3x - 9$.

(**7**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -2x - 1 - 2y &= -6x + 5 - y, \text{ so} \\ -2y + y &= -6x + 2x + 5 + 1 \\ -y &= -4x + 6 \\ y &= 4x - 6 \end{aligned}$$

Hence, the gradient of the original line is $m = 4$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $\quad y = 4x + c \quad$ and we can substitute the coordinates of the point $(x_1, y_1) = (-2, -10)$ into this equation to get the value for c.
$-10 = 4 \times (-2) + c$, so $-10 = -8 + c$. Hence $c = -10 - (-8) = -2$.
Hence the equation of the line is $\quad y = 4x - 2$.

(8) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 0 &= 10y - 10x - 40, \text{ so} \\ -10y &= -10x - 40 \\ y &= x + 4 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 1$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\dfrac{1}{m_0}$. Hence $m = -1$.
Thus the equation of the line is $\quad y = -x + c \quad$ and we can substitute the coordinates of the point $(x_1, y_1) = (-6, 6)$ into this equation to get the value of c:
$6 = 6 + c$. Hence $c = 6 - 6 = 0$.
Hence the equation of the line is $\quad y = -x$.

(9) To determine whether the given line passes through the point $(x_1, y_1) = (4, -10)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} -3x &= -y + 4, \text{ so} \\ -3 \times 4 &= 10 + 4 \\ -12 &= 10 + 4 \\ -12 &= 14 \end{aligned}$$

The last statement is **not true**, so our line **does not** pass through the point $(4, -10)$.

(10) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 0 &= -15 + 5y, \text{ so} \\ -5y &= -15 \\ y &= 3 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(1, -5)$ lies on the new line, so the equation of the new line is $\quad x = 1$.

(11) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 3y &= -21, \text{ so} \\ y &= -7 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $\quad y = c \quad$ and we can substitute the coordinates of the point $(x_1, y_1) = (-3, -1)$ into this equation to get the value for c.
$-1 = c$.
Hence the equation of the line is $\quad y = -1$.

(12) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(-2, 10)$ lies on the new line, so the equation of the new line is $\quad y = 10$.

(13) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(3, 4)$ lies on the new line, so the equation of the new line is $\quad x = 3$.

2. **(1)** Let $(x_1, y_1) = (-7, -2)$ and $(x_2, y_2) = (-2, -1)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(-7-(-2))^2 + (-2-(-1))^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$.
Hence $d = \sqrt{26}$

(2) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -4x &= 3y - 5, \quad \text{so} \\ -3y &= 4x - 5 \\ y &= -\frac{4}{3}x + \frac{5}{3} \end{aligned}$$

Hence the gradient is $m = -\frac{4}{3}$ and the y-intercept is $c = \frac{5}{3}$.

(3) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 6y - 1 - 2x &= 3y - 4x + 2, \text{ so} \\ 6y - 3y &= -4x + 2x + 2 + 1 \\ 3y &= -2x + 3 \\ y &= -\frac{2}{3}x + 1 \end{aligned}$$

Hence the gradient is $m = -\frac{2}{3}$ and the y-intercept is $c = 1$.

(4) Thus the equation of the line is $y = 5x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (3, 10)$ into this equation to get the value for c. Hence $10 = 5 \times 3 + c$, so $-5 = c$.
Hence the equation of the line is $y = 5x - 5$.

(5) Let $(x_1, y_1) = (-9, 9)$ and $(x_2, y_2) = (-3, -3)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.
Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-9}{-3-(-9)} = \frac{-12}{6}$. Hence $m = -2$.
Thus the equation of the line is $y = -2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-9, 9)$ into this equation to get the value for c.
Hence $9 = -2 \times (-9) + c$, so $9 = 18 + c$. Hence $c = 9 - 18 = -9$.
Hence the equation of the line is $y = -2x - 9$.

(6) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 8y &= -32x + 40, \text{ so} \\ y &= -4x + 5 \end{aligned}$$

Hence, the gradient of the original line is $m = -4$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = -4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (1, -11)$ into this equation to get the value for c.
$-11 = -4 \times 1 + c$, so $-11 = -4 + c$. Hence $c = -11 - (-4) = -7$.
Hence the equation of the line is $y = -4x - 7$.

(7) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -5x - 4y &= y - 30 - 5x, \text{ so} \\ -4y - y &= -5x + 5x - 30 \\ -5y &= -30 \\ y &= 6 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation

of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (9, -9)$ into this equation to get the value for c.
$-9 = c$.
Hence the equation of the line is $y = -9$.

(**8**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$7 = -4x - y, \text{ so}$$
$$y = -4x - 7$$

Hence the gradient of the original line is $m_0 = -4$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\dfrac{1}{m_0}$. Hence $m = \dfrac{1}{4}$.
Thus the equation of the line is $y = \dfrac{1}{4}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-28, -4)$ into this equation to get the value of c:
$-4 = \dfrac{1}{4} \times (-28) + c$, so $-4 = -7 + c$. Hence $c = -4 - (-7) = 3$.
Hence the equation of the line is $y = \dfrac{1}{4}x + 3$.

(**9**) To determine whether the given line passes through the point $(x_1, y_1) = (-5, -4)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$-y + 2x = -6, \text{ so}$$
$$4 + 2 \times (-5) = -6$$
$$4 - 10 = -6$$
$$-6 = -6$$

The last statement is **true**, so our line **does** pass through the point $(-5, -4)$.

(**10**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$70 = 10y, \text{ so}$$
$$-10y = -70$$
$$y = 7$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(-1, -7)$ lies on the new line, so the equation of the new line is $x = -1$.

(**11**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$4y = 32, \text{ so}$$
$$y = 8$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-5, 4)$ into this equation to get the value for c.
$4 = c$.
Hence the equation of the line is $y = 4$.

(**12**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(-1, -3)$ lies on the new line, so the equation of the new line is $y = -3$.

(**13**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(-7, -9)$ lies on the new line, so the equation of the new line is $x = -7$.

3. (**1**) Let $(x_1, y_1) = (-5, 2)$ and $(x_2, y_2) = (0, -7)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(-5-0)^2 + (2-(-7))^2} = \sqrt{(-5)^2 + 9^2} = \sqrt{25+81} = \sqrt{106}$.
Hence $d = \sqrt{106}$

(**2**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -1 &= 2y + 2x, \quad \text{so} \\ -2y &= 2x + 1 \\ y &= -x - \frac{1}{2} \end{aligned}$$

Hence the gradient is $m = -1$ and the y-intercept is $c = -\frac{1}{2}$.

(**3**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -y - 7 - 3x &= 3y + 9x - 8, \text{ so} \\ -y - 3y &= 9x + 3x - 8 + 7 \\ -4y &= 12x - 1 \\ y &= -3x + \frac{1}{4} \end{aligned}$$

Hence the gradient is $m = -3$ and the y-intercept is $c = \frac{1}{4}$.

(**4**) Thus the equation of the line is $y = -4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (6, 4)$ into this equation to get the value for c. Hence $4 = -4 \times 6 + c$, so $28 = c$.
Hence the equation of the line is $y = -4x + 28$.

(**5**) Let $(x_1, y_1) = (-5, -8)$ and $(x_2, y_2) = (-5, -3)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.
Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-8)}{-5-(-5)} = \frac{5}{0}$.
Therefore this line has an infinite gradient, and is parallel to the y-axis. It's equation is of the form $x = k$, where k is a constant.
Hence the equation of the line is $x = -5$.

(**6**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -36 + 18x &= -6y, \text{ so} \\ 6y &= -18x + 36 \\ y &= -3x + 6 \end{aligned}$$

Hence, the gradient of the original line is $m = -3$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = -3x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (4, -21)$ into this equation to get the value for c.
$-21 = -3 \times 4 + c$, so $-21 = -12 + c$. Hence $c = -21 - (-12) = -9$.
Hence the equation of the line is $y = -3x - 9$.

(**7**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 2y + 2 + 4x &= y - 7 + x, \text{ so} \\ 2y - y &= x - 4x - 7 - 2 \\ y &= -3x - 9 \end{aligned}$$

Hence, the gradient of the original line is $m = -3$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation

of the line is $y = -3x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-9, 37)$ into this equation to get the value for c.
$37 = -3 \times (-9) + c$, so $37 = 27 + c$. Hence $c = 37 - 27 = 10$.
Hence the equation of the line is $y = -3x + 10$.

(**8**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -36 - 12x &= -4y, \text{ so} \\ 4y &= 12x + 36 \\ y &= 3x + 9 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 3$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = -\frac{1}{3}$.
Thus the equation of the line is $y = -\frac{1}{3}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (30, -5)$ into this equation to get the value of c:
$-5 = -\frac{1}{3} \times 30 + c$, so $-5 = -10 + c$. Hence $c = -5 - (-10) = 5$.
Hence the equation of the line is $y = -\frac{1}{3}x + 5$.

(**9**) To determine whether the given line passes through the point $(x_1, y_1) = (-7, -6)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} 0 &= -y + 7 - 7x, \text{ so} \\ 0 &= 6 + 7 - 7 \times (-7) \\ 0 &= 6 + 7 + 49 \\ 0 &= 62 \end{aligned}$$

The last statement is **not true**, so our line **does not** pass through the point $(-7, -6)$.

(**10**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -8 &= -4y, \text{ so} \\ 4y &= 8 \\ y &= 2 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(7, 2)$ lies on the new line, so the equation of the new line is $x = 7$.

(**11**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 0 &= 10y - 60, \text{ so} \\ -10y &= -60 \\ y &= 6 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-2, 4)$ into this equation to get the value for c.
$4 = c$.
Hence the equation of the line is $y = 4$.

(**12**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(9, -9)$ lies on the new line, so the equation of the new line is $y = -9$.

(**13**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(4, 9)$ lies on the new line, so the equation of the new line is $\quad x = 4$.

4. (**1**) Let $(x_1, y_1) = (-5, 7)$ and $(x_2, y_2) = (-5, -6)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(-5-(-5))^2 + (7-(-6))^2} = \sqrt{0^2 + 13^2} = \sqrt{0 + 169} = \sqrt{169}$.
Hence $d = 13$

(**2**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 4y &= 2x - 7, \quad \text{so} \\ y &= \frac{1}{2}x - \frac{7}{4} \end{aligned}$$

Hence the gradient is $m = \dfrac{1}{2}$ and the y-intercept is $c = -\dfrac{7}{4}$.

(**3**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -7y - x + 7 &= -6y + 1 + 6x, \text{ so} \\ -7y + 6y &= 6x + x + 1 - 7 \\ -y &= 7x - 6 \\ y &= -7x + 6 \end{aligned}$$

Hence the gradient is $m = -7$ and the y-intercept is $c = 6$.

(**4**) Thus the equation of the line is $y = -1x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-4, -3)$ into this equation to get the value for c. Hence $-3 = -1 \times (-4) + c$, so $-7 = c$.
Hence the equation of the line is $y = -x - 7$.

(**5**) Let $(x_1, y_1) = (3, -7)$ and $(x_2, y_2) = (0, -4)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.
Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-7)}{0 - 3} = \dfrac{3}{-3}$. Hence $m = -1$.
Thus the equation of the line is $y = -x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (3, -7)$ into this equation to get the value for c.
Hence $-7 = -1 \times 3 + c$, so $-7 = -3 + c$. Hence $c = -7 - (-3) = -4$.
Hence the equation of the line is $y = -x - 4$.

(**6**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 10y + 40x &= 0, \text{ so} \\ 10y &= -40x \\ y &= -4x \end{aligned}$$

Hence, the gradient of the original line is $m = -4$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $\quad y = -4x + c \quad$ and we can substitute the coordinates of the point $(x_1, y_1) = (3, -21)$ into this equation to get the value for c.
$-21 = -4 \times 3 + c$, so $-21 = -12 + c$. Hence $c = -21 - (-12) = -9$.
Hence the equation of the line is $\quad y = -4x - 9$.

(**7**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 4x - 7y + 1 &= 37 - y - 20x, \text{ so} \\ -7y + y &= -20x - 4x + 37 - 1 \\ -6y &= -24x + 36 \\ y &= 4x - 6 \end{aligned}$$

Hence, the gradient of the original line is $m = 4$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = 4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-4, -17)$ into this equation to get the value for c.
$-17 = 4 \times (-4) + c$, so $-17 = -16 + c$. Hence $c = -17 - (-16) = -1$.
Hence the equation of the line is $y = 4x - 1$.

(**8**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -49 &= 21x - 7y, \text{ so} \\ 7y &= 21x + 49 \\ y &= 3x + 7 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 3$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = -\frac{1}{3}$.
Thus the equation of the line is $y = -\frac{1}{3}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (9, -2)$ into this equation to get the value of c:
$-2 = -\frac{1}{3} \times 9 + c$, so $-2 = -3 + c$. Hence $c = -2 - (-3) = 1$.
Hence the equation of the line is $y = -\frac{1}{3}x + 1$.

(**9**) To determine whether the given line passes through the point $(x_1, y_1) = (6, 57)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} 60x - 18 &= 6y, \text{ so} \\ 60 \times 6 - 18 &= 6 \times 57 \\ 360 - 18 &= 342 \\ 342 &= 342 \end{aligned}$$

The last statement is **true**, so our line **does** pass through the point $(6, 57)$.

(**10**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 4y &= -12, \text{ so} \\ y &= -3 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(-4, -3)$ lies on the new line, so the equation of the new line is $x = -4$.

(**11**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -5y &= 0, \text{ so} \\ y &= 0 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (3, 10)$ into this equation to get the value for c.
$10 = c$.
Hence the equation of the line is $y = 10$.

(**12**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(-7, 6)$ lies on the new line, so the equation of the new line is $y = 6$.

(**13**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(-2, 2)$ lies on the new line, so the equation of the new line is $x = -2$.

5. (**1**) Let $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (5, 7)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(3-5)^2 + (1-7)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{4 + 36} = \sqrt{40}$.
Hence $d = 2\sqrt{10}$

(**2**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} -9 - 5y &= -6x, \quad \text{so} \\ -5y &= -6x + 9 \\ y &= \frac{6}{5}x - \frac{9}{5} \end{aligned}$$

Hence the gradient is $m = \frac{6}{5}$ and the y-intercept is $c = -\frac{9}{5}$.

(**3**) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 4 - 5y - 8x &= y + 7x + 3, \text{ so} \\ -5y - y &= 7x + 8x + 3 - 4 \\ -6y &= 15x - 1 \\ y &= -\frac{5}{2}x + \frac{1}{6} \end{aligned}$$

Hence the gradient is $m = -\frac{5}{2}$ and the y-intercept is $c = \frac{1}{6}$.

(**4**) Thus the equation of the line is $y = 5x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (9, -1)$ into this equation to get the value for c. Hence $-1 = 5 \times 9 + c$, so $-46 = c$.
Hence the equation of the line is $y = 5x - 46$.

(**5**) Let $(x_1, y_1) = (1, 9)$ and $(x_2, y_2) = (-7, -3)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.
Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 9}{-7 - 1} = \frac{-12}{-8}$. Hence $m = \frac{3}{2}$.
Thus the equation of the line is $y = \frac{3}{2}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (1, 9)$ into this equation to get the value for c.
Hence $9 = \frac{3}{2} \times 1 + c$, so $9 = \frac{3}{2} + c$. Hence $c = 9 - \frac{3}{2} = \frac{15}{2}$.
Hence the equation of the line is $y = \frac{3}{2}x + \frac{15}{2}$.

(**6**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -5y - 5 &= -10x, \text{ so} \\ -5y &= -10x + 5 \\ y &= 2x - 1 \end{aligned}$$

Hence, the gradient of the original line is $m = 2$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = 2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-7, -18)$ into this equation to get the value for c.
$-18 = 2 \times (-7) + c$, so $-18 = -14 + c$. Hence $c = -18 - (-14) = -4$.
Hence the equation of the line is $y = 2x - 4$.

(**7**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 6y + 7x + 1 &= -3y + 16x + 37, \text{ so} \\ 6y + 3y &= 16x - 7x + 37 - 1 \\ 9y &= 9x + 36 \\ y &= x + 4 \end{aligned}$$

Hence, the gradient of the original line is $m = 1$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-7, -12)$ into this equation to get the value for c.
$-12 = 1 \times (-7) + c$, so $-12 = -7 + c$. Hence $c = -12 - (-7) = -5$.
Hence the equation of the line is $y = x - 5$.

(**8**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -32 + 16x &= 4y, \text{ so} \\ -4y &= -16x + 32 \\ y &= 4x - 8 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 4$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = -\frac{1}{4}$.
Thus the equation of the line is $y = -\frac{1}{4}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (28, -4)$ into this equation to get the value of c:
$-4 = -\frac{1}{4} \times 28 + c$, so $-4 = -7 + c$. Hence $c = -4 - (-7) = 3$.
Hence the equation of the line is $y = -\frac{1}{4}x + 3$.

(**9**) To determine whether the given line passes through the point $(x_1, y_1) = (-7, -40)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} 6y &= -30 + 30x, \text{ so} \\ 6 \times (-40) &= -30 + 30 \times (-7) \\ -240 &= -30 - 210 \\ -240 &= -240 \end{aligned}$$

The last statement is **true**, so our line **does** pass through the point $(-7, -40)$.

(**10**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -18 + 2y &= 0, \text{ so} \\ 2y &= 18 \\ y &= 9 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(-9, 10)$ lies on the new line, so the equation of the new line is $x = -9$.

(**11**) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 4y &= -4, \text{ so} \\ y &= -1 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation

of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (5, 8)$ into this equation to get the value for c.
$8 = c$.
Hence the equation of the line is $y = 8$.

(**12**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(9, 1)$ lies on the new line, so the equation of the new line is $y = 1$.

(**13**) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(8, 4)$ lies on the new line, so the equation of the new line is $x = 8$.