MATH1040/7040 Assignment 2 SOLUTIONS

1.

(a) $-5z\,(-6+2z) = -6\times(-5z) + 2z\times(-5z) = 30z - 10z^2$

(b) $(-6+7x)\,(3-6x) = -6\times 3 - 6\times(-6x) + 7x\times 3 + 7x\times(-6x) = -18 + 36x + 21x - 42x^2 = -42x^2 + 57x - 18$

(c) Substituting for z into the equation gives $2 = 4x - 3$, so $4x = 2+3$, so $4x = 5$, so $\dfrac{4x}{4} = \dfrac{5}{4}$

Hence $x = \dfrac{5}{4}$

(d) $-4x - 6 = 0$, so $-4x = 6$, so $\dfrac{-4x}{-4} = \dfrac{6}{-4}$

Hence $x = -\dfrac{3}{2}$

(e) $\dfrac{-4x}{2} - 6 = 2$, so $-2x = 2+6$, so $-2x = 8$, so $\dfrac{-2x}{-2} = \dfrac{8}{-2}$

Hence solution is: $x = -4$

(f) $-4 + \dfrac{-4}{-4x} = 2$, so $\dfrac{1}{x} = 4+2$, so $\dfrac{1}{x} = 6$, so $1 = 6x$, so $x = \dfrac{1}{6}$

Hence solution is: $x = \dfrac{1}{6}$

(g)

$$\begin{aligned}\frac{-11}{3}\times\frac{13}{5} &= \frac{-11\times 13}{3\times 5}\\ &= \frac{-143}{15}\\ &= -9\frac{8}{15}\end{aligned}$$

Hence solution is: $z{=}{-9}\dfrac{8}{15}$

(h) $6 = 4z + 4$, so $6 - 4 = 4z$, so $2 = 4z$, so $\dfrac{2}{4} = \dfrac{4z}{4}$

Hence $z = \dfrac{1}{2}$

(i) $|5x+5| = 1$, so

$$\begin{aligned}&5x+5 = 1 &\quad \textit{or} \quad& 5x+5 = -1\\ &5x = 1-5 && 5x = -1-5\\ &5x = -4 && 5x = -6\\ &\frac{5x}{5} = \frac{-4}{5} && \frac{5x}{5} = \frac{-6}{5}\end{aligned}$$

Hence the solutions are: $x = -\dfrac{4}{5}$ and $x = -\dfrac{6}{5}$

(j) Since the two integers are consecutive, we know that there is a difference of one between them. Let the smaller integer be represented by n, so the larger integer will then be $(n+1)$. We then have:

$$\begin{aligned} n+(n+1) &= 15 \\ \Longrightarrow \quad 2\times n+1 &= 15 \\ \Longrightarrow \quad 2\times n &= 14 \\ \Longrightarrow \quad n &= 7 \end{aligned}$$

Note that this gives us the value of the *lower* integer only! We need *both* integers!

So if the smaller number is 7, then the larger number must be 8.

(k) $\sqrt{128x} = 4\sqrt{8}$, so $\sqrt{128x} = \sqrt{4\times 4\times 8} = \sqrt{128}$, so $128x = 128$. Hence $x = 1$

(l) $\sqrt{45} = x\sqrt{5}$. Now $\sqrt{45} = \sqrt{9\times 5} = \sqrt{3\times 3\times 5} = 3\sqrt{5}$. Hence $x = 3$

(m)

$$\begin{aligned} \sqrt{2}\left(\sqrt{5}+\sqrt{4}\right) &= \sqrt{2}\times\sqrt{5}+\sqrt{2}\times\sqrt{4} \\ &= \sqrt{2\times 5}+\sqrt{2\times 4} \\ &= \sqrt{10}+\sqrt{8} \\ &= \sqrt{10}+2\sqrt{2} \end{aligned}$$

(n)

$$\begin{aligned} \left(\sqrt{6}+\sqrt{6}\right)\left(\sqrt{8}+\sqrt{6}\right) &= \sqrt{6}\times\sqrt{8}+\sqrt{6}\times\sqrt{6}+\sqrt{6}\times\sqrt{8}+\sqrt{6}\times\sqrt{6} \\ &= \sqrt{6\times 8}+\sqrt{6\times 6}+\sqrt{6\times 8}+\sqrt{6\times 6} \\ &= \sqrt{48}+6+\sqrt{48}+6 \\ &= 4\sqrt{3}+6+4\sqrt{3}+6 \\ &= 6+6+4\sqrt{3}+4\sqrt{3} \\ &= 12+8\sqrt{3} \end{aligned}$$

2.

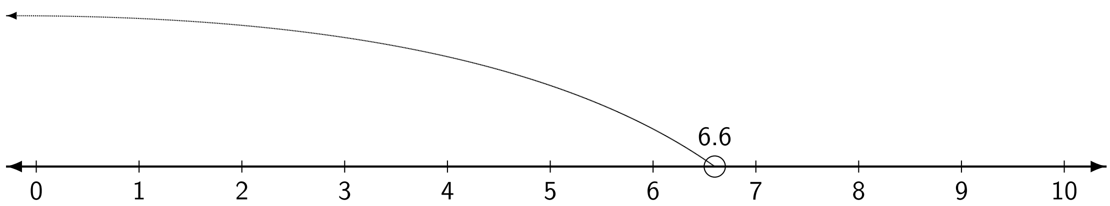
(a)

$$\begin{aligned} y^{-2}y^{-3}x^{2}y^{-3}\div x^{-1}\times y^{-2} &= y^{-2}y^{-3}x^{2}y^{-3}\times x^{1}\times y^{-2} \\ &= x^{2}x^{1}y^{-2}y^{-3}y^{-3}y^{-2} \\ &= x^{2+1}y^{-2-3-3-2} \\ &= x^{3}y^{-10} \end{aligned}$$

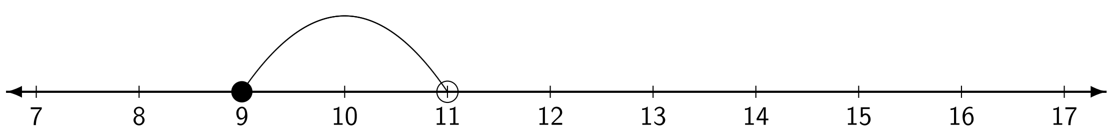
(b) $\dfrac{-5y^4y^4}{y^{-3}y^{-5}} = \dfrac{-5y^{4+4}}{y^{-3-5}} = \dfrac{-5y^8}{y^{-8}} = -5y^{8-(-8)} = -5y^{16}$

3.

(a) In interval form the answer is $(-\infty, 6.6)$ and on a real line the answer is:



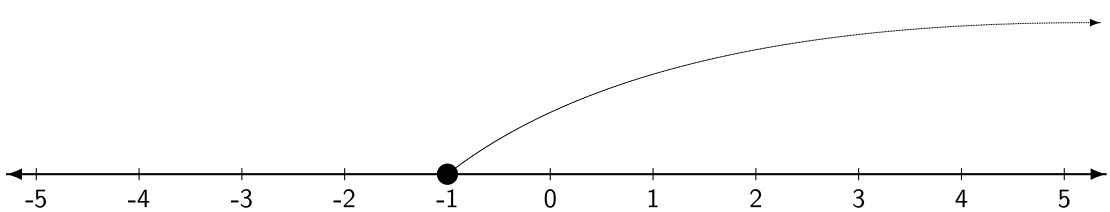
(b) In inequality form the answer is $9 \leq x < 11$ and on a real line the answer is:



(c)

$$
\begin{aligned}
8x - 6 &\geq 4x - 10 \\
8x - 6 + 6 &\geq 4x - 10 + 6 \\
8x &\geq 4x - 4 \\
8x - 4x &\geq 4x - 4x - 4 \\
4x &\geq -4 \\
4x \div 4 &\geq -4 \div 4 \\
x &\geq -1
\end{aligned}
$$

In interval format the answer is $[-1, \infty)$, and on a real line the answer is:



4. Mayumi ate x pieces of sushi. Rumi ate 4 more, so $x + 4$.
So, $x + x + 4 = 26$
$2x = 22$
$x = 11$

So Mayumi ate 11 pieces and Rumi ate 11 + 4 = 15 pieces (check: 11 + 15 = 26)

5. Let the first hospital have x doctors. The second hospital therefore has $3x - 20$ doctors.
So, $x + 3x - 20 = 204$
$4x = 224$
$x = 56$

So the first hospital has 56 doctors and the second hospital has $56 + 3 \times 56 - 20 = 148$. (check: $56 + 148 = 204$)

6. $((x + x^2) \div x - 16 - x) \div 3 = \left(\frac{x + x^2}{x} - 16 - x\right) \div 3$

$= \left(\frac{x(1 + x)}{x} - 16 - x\right) \div 3$

$= (1 + x - 16 - x) \div 3$

$= -15 \div 3$

$= -5$

The x's disappear, so regardless of what number x is the answer is always -5.