MATH1040/7040 ASSIGNMENT 3 SOLUTIONS

1. (a)

$$\sum_{i=x-1}^{x+1} -i = -3$$

$$-(x-1) - x - (x+1) = -3$$

$$-3x = -3$$

Hence $x = 1$

(b) $\sum_{i=0}^{3} (-1)^i i = (-1)^0 \times 0 + (-1)^1 \times 1 + (-1)^2 \times 2 + (-1)^3 \times 3 = 0 - 1 + 2 - 3 = -2$

(c) $\sum_{i=3}^{8} xi = 132,$ so $3x + 4x + 5x + 6x + 7x + 8x = 132,$ so $33x = 132$

Hence $x = 4$

(d) $\sum_{i=-2}^{1} -2x = 8,$ so $-2x - 2x - 2x - 2x = 8,$ so $-8x = 8$

Hence $x = -1$

(e) $-\frac{6}{4} - \frac{6}{5} - \frac{6}{6} - \frac{6}{7} = \sum_{k=4}^{7} \frac{-6}{k}$

(f) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 1 - 7y - 10x &= 9 + 9y + 3x, \text{ so} \\ -7y - 9y &= 3x + 10x + 9 - 1 \\ -16y &= 13x + 8 \\ y &= -\frac{13}{16}x - \frac{1}{2} \end{aligned}$$

Hence the gradient is $m = -\frac{13}{16}$ and the y-intercept is $c = -\frac{1}{2}$.

(g) Let $(x_1, y_1) = (9, -9)$ and $(x_2, y_2) = (5, -10)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - (-9)}{5 - 9} = \frac{-1}{-4}$. Hence $m = \frac{1}{4}$.

Thus the equation of the line is $y = \frac{1}{4}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (9, -9)$ into this equation to get the value for c.

Hence $-9 = \frac{1}{4} \times 9 + c$, so $-9 = \frac{9}{4} + c$. Hence $c = -9 - \frac{9}{4} = -\frac{45}{4}$.

Hence the equation of the line is $y = \frac{1}{4}x - \frac{45}{4}$.

(h) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 10 - 6x - 8y &= y + 30x - 44, \text{ so} \\ -8y - y &= 30x + 6x - 44 - 10 \\ -9y &= 36x - 54 \\ y &= -4x + 6 \end{aligned}$$

Hence, the gradient of the original line is $m = -4$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = -4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-6, 33)$ into this equation to get the value for c.
$33 = -4 \times (-6) + c$, so $33 = 24 + c$. Hence $c = 33 - 24 = 9$.
Hence the equation of the line is $y = -4x + 9$.

(i) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -y &= -1, \text{ so} \\ y &= 1 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (10, -1)$ into this equation to get the value for c.
$-1 = c$.
Hence the equation of the line is $y = -1$.

(j) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 5y &= 5x, \text{ so} \\ y &= x \end{aligned}$$

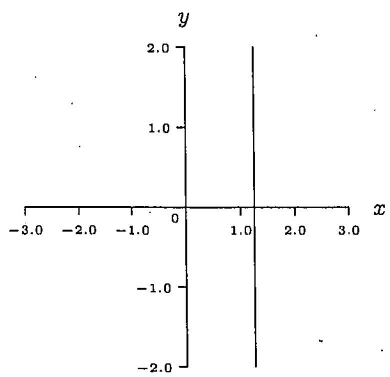
Hence the gradient of the original line is $m_0 = 1$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = -1$.
Thus the equation of the line is $y = -x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (0, 3)$ into this equation to get the value of c:
$3 = c$.
Hence the equation of the line is $y = -x + 3$.

(k) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(-8, 8)$ lies on the new line, so the equation of the new line is $y = 8$.

2. (a) First we rearrange the equation to get $x = \frac{5}{4}$. Therefore, $x = \frac{5}{4}$ regardless of the value of y. Hence, the line does not intercept the y-axis at all and there is no y-intercept.

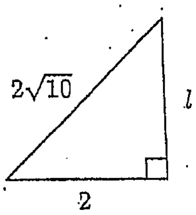
(b) The line $x = \frac{5}{4}$ has constant x-value. Hence, the x-intercept is $x = \frac{5}{4}$.

(c) (Note that the scaling of the axes on the graph below are not equal.)



3.

(a) $(2\sqrt{10})^2 = 2^2 + l^2 \Rightarrow 40 = 4 + l^2 \Rightarrow l^2 = 36 \Rightarrow l = 6$. Window is 7m high, so he cannot reach.



(b) $y = 2x + c$, $(0, 0)$ is on the line $\Rightarrow$ $0 = 2 \times 0 + c \Rightarrow$ $c = 0 \Rightarrow$ $y = 2x$

(c) We know the equation of the ladder is $y = 2x$. When $x = 2\sqrt{2}$, $y = 2x = 2 \times 2\sqrt{2} = 4\sqrt{2} \Rightarrow$ window is $4\sqrt{2}$m high.

(d) When they have travelled half of the way down the ladder, their point must have an x-coordinate of $\sqrt{2}$. Hence the equation of the vertical line through which they fall is $x = \sqrt{2}$