MATH1040/7040 ASSIGNMENT 4 SOLUTIONS

1. (a) Let $(x_1, y_1) = (10, \sqrt{3})$ and $(x_2, y_2) = (-6, \sqrt{3})$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(10 - (-6))^2 + \left(\sqrt{3} - \sqrt{3}\right)^2} = \sqrt{16^2 + 0^2} = \sqrt{256 + 0} = \sqrt{256}$.
Hence $d = 16$

(b) First we number the equations for convenience:

$$
\begin{aligned}
5y - 9x &= -50 \qquad (1)\\
-40y + 72x &= 404 \qquad (2)
\end{aligned}
$$

It's probably easier to solve these using elimination. Multiply equation (1) by 8 , giving

$$
\begin{aligned}
40y - 72x &= -400 \qquad (3)\\
-40y + 72x &= 404 \qquad (4)
\end{aligned}
$$

We add both sides of equations (3) and (4) , giving

$$40y - 40y - 72x + 72x = -400 + 404 \qquad (5)$$

Simplifying equation (5) gives

$$0 = 4 \qquad (6)$$

Statement (6) is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.

(c) First we number the equations for convenience:

$$
\begin{aligned}
-12 - 8y &= -2x \qquad (1)\\
-5x &= -214 + 3y \qquad (2)
\end{aligned}
$$

We solve these using substitution. Dividing both sides of equation (1) by -2 gives

$$6 + 4y = x \qquad (3)$$

Substituting for x in equation (2),

$$-5 \times (6 + 4y) = -214 + 3y \qquad (4)$$

Now (4) is an equation only involving y which gives:

$$
\begin{aligned}
-30 - 20y &= -214 + 3y\\
-23y &= -184\\
y &= 8
\end{aligned}
$$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = 6 + 4 \times 8 = 38$$

Hence the simultaneous solution to equations (1) and (2) is $(38, 8)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$
\begin{aligned}
(1)\quad & -12 - 8 \times 8 = -2 \times 38 & \qquad (2)\quad & -5 \times 38 = -214 + 3 \times 8\\
& -12 - 64 = -76 & & -190 = -214 + 24\\
& -76 = -76 & & -190 = -190
\end{aligned}
$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(d) $f(w) = 3\,(w+3)^2$

When determining the domain of this function, we need to keep in mind the following:

* $*$ there are no square roots or absolute value signs;
* $*$ we can square any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of w can be substituted into f .

(e) $f(w) = \dfrac{7}{w^2 - 5}$

When determining the domain of this function, we need to keep in mind the following:

* $*$ denominator of a fraction cannot be 0, so $w^2 - 5 \neq 0$;
* $*$ so $w^2 \neq 5$;
* $*$ we can square any number and result will always be a positive number or 0, so $w \neq \pm\sqrt{5}$.

Hence, the domain of this function is $(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)$, i.e. $w \neq \pm\sqrt{5}$.

(f) $f(w) = \sqrt{5\,|w|}$

When evaluating the range, we need to keep in mind the following (starting with variable w):

* $*$ absolute value is always positive or 0, so $|w| \geq 0$;
* $*$ square root is always positive or 0, so $\sqrt{5\,|w|} \geq 0$.

Hence, the range of this function is $[0, \infty)$.

(g) $f(x) = \dfrac{-11}{\sqrt{x} + 1}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

* $*$ square root is always positive or 0, so $0 \leq \sqrt{x}$;
* $*$ fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
* $*$ so $1 \leq \sqrt{x} + 1$.

Hence, the range of this function is $[-11, 0)$.

(h) $f(w) = \dfrac{9}{\sqrt{|w|}}$

When determining the domain of this function, we need to keep in mind the following:

* $*$ denominator of a fraction cannot be 0, so $\sqrt{|w|} \neq 0$;
* $*$ we can only take the square root of positive numbers or 0, so $|w| > 0$;
* $*$ we can find the absolute value of any number.

Hence, the domain of this function is $(-\infty, 0) \cup (0, \infty)$, i.e. $w \neq 0$.

When evaluating the range, we need to keep in mind the following (starting with variable w):

* $*$ absolute value is always positive or 0, so $|w| \geq 0$;
* $*$ square root is always positive or 0, so $\sqrt{|w|} \geq 0$;
* $*$ fraction can be 0 only if numerator is 0, so $\dfrac{9}{\sqrt{|w|}} > 0$.

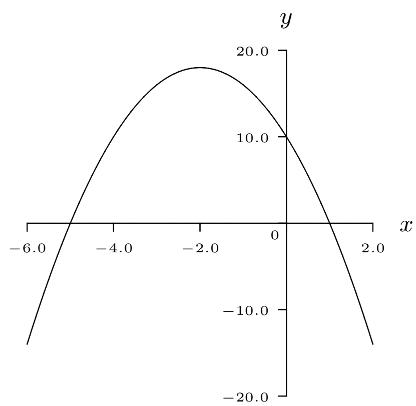
Hence, the range of this function is $(0, \infty)$.

2. (a) The roots of $y = -2x^2 - 8x + 10$ are the x values that satisfy $-2x^2 - 8x + 10 = 0$. You can solve this equation either by using the quadratic formula or by factoring. Here we will use factoring.

First divide through by -2 to get $x^2 + 4x - 5 = 0$. Now because $x^2 + 4x - 5 = (x + 5)(x - 1)$, the two roots of the quadratic equation are $x = -5, 1$.

(b) The y-intercept occurs when $x = 0$, so substituting this into $y = -2x^2 - 8x + 10$ gives $y = 10$.

(c)



3. $3x^2 + 3x - 36 = 0$, so we use $a = 3, b = 3, c = -36$ in the quadratic formula. Hence

$$\begin{aligned}
x &= \frac{-3 \pm \sqrt{3^2 - 4 \times 3 \times (-36)}}{2 \times 3} \\
&= \frac{-3 \pm \sqrt{9 - (-432)}}{6} \\
&= \frac{-3 \pm \sqrt{441}}{6} \\
&= \frac{-3 + 21}{6} \quad or \quad \frac{-3 - 21}{6} \\
&= \frac{18}{6} \quad or \quad \frac{-24}{6} \\
&= 3 \quad or \quad -4
\end{aligned}$$

4. BONUS QUESTION
$\frac{44}{7}$ clubs.