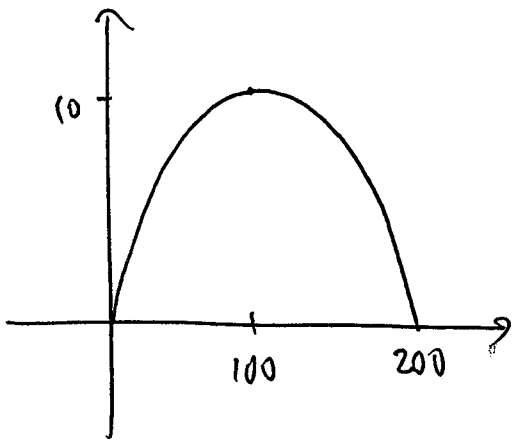
MATH 1040/7040 ASSIGNMENT 6 SOLUTIONS

1.



$y = ax^2 + bx + c$

$(0,0) \Rightarrow \quad c = 0 \quad$ — (1)

$(100,10) \Rightarrow \quad 10 = 100^2 a + 100b$ — (2)

$(200,0) \Rightarrow \quad 0 = 200^2 a + 200b$ — (3)

(2) $\quad 10000a + 100b = 10$

$\Rightarrow \quad 1000a + 10b = 1$ — (4)

(3) $\quad 40000a + 200b = 0$

$\Rightarrow \quad 200a + b = 0$ — (5)

(5) $\times 5 \quad 1000a + 5b = 0$

(5) − (6) $\quad 1000a + 5b = 0$

$\quad - \quad 1000a + 10b = 1$

$-5b = -1$

$b = \frac{1}{5} = 0.2$

$\therefore \quad a = -0.001$

$\therefore \quad y = -0.001x^2 + 0.2x$

2. $(x-a)^2+(y-b)^2=r^2$

$(1,4)$, $(0,3)$ circle centred on x-axis, so $b=0$ e

$(x-a)^2+y^2=r^2$ is the equation.

sub in $(1,4)$ $\quad (1-a)^2+4^2=r^2$

$1-2a+a^2+16=r^2$

$a^2-2a+17=r^2$ —①

Sub in $(0,3)$ $\quad (0-a)^2+3^2=r^2$

$a^2+9=r^2$ —②

① $r^2=a^2-2a+17$

② $r^2=a^2+9$

$\therefore a^2-2a+17=a^2+9$

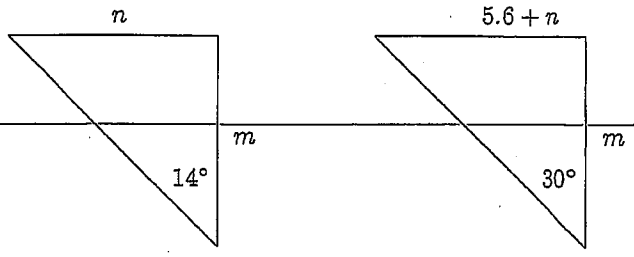
$\therefore -2a=-8$

$\therefore a=4$

$\therefore r=5$

$\therefore$ Equation is $(x-4)^2+y^2=5^2$ (25)

3 (a) We have two triangles representing the limits of the angle at which he can kick. The left triangle corresponds to kicking at 14°, and the right corresponds to kicking at 30°.



From the diagrams,

$\tan 14° \approx \frac{1}{4} = \frac{n}{m}$ (1)

$\tan 30° \approx \frac{1}{2} = \frac{5.6+n}{m}$ (2)

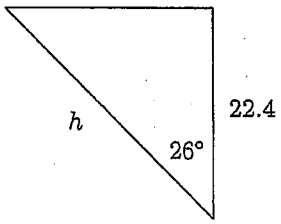
From (1) we have $4n = m$ (3)

From (2) we have $2(5.6+n) \approx m \Rightarrow m \approx 11.2 + 2n$ (4)

Substitute (3) into (4) $\Rightarrow 4n \approx 11.2 + 2n \Rightarrow n \approx 5.6$ metres. Substitute into (3) $\Rightarrow m \approx 22.4$ metres.

(b) Let a be the angle, so we have $\tan a = \frac{n+2.8}{m} = \frac{5.6+2.8}{22.4} = 0.375$, so $a \approx 20.556°$. $\approx 21°$

(c) $\cos 26° \approx 0.9 = \frac{22.4}{h} \Rightarrow 0.9h \approx 22.4 \Rightarrow h \approx \frac{22.4}{0.9} \approx 24.9$



So the kick is good.

The Barbarians win!

4. $1000 20% pa

a) $F = P(1+r)^x$

$= 1000(1+0.2)^{30}$

$\approx \underline{\$237,376.31}$. Wow!

b) $F = Pe^{rx}$

$= 1000\, e^{0.2 \times 30}$

$\approx \underline{\$403,428.79}$. Even better!

c) $403,428.79 \approx Pe^{0.2 \times 10}$

$\therefore \underline{P \approx \$54,598.15}$

d) $403428.79 \approx Pe^{0.3 \times 10}$

$\therefore \underline{P \approx \$20085.54}$.

5. <u>BONUS QUESTION</u>

$y = 2x^2 - 4x + c$

$b^2 - 4ac = 0$ to touch x-axis once,

so $(-4)^2 - 4 \times 2 \times c = 0$

$16 - 8c = 0$

$\therefore \underline{c = 2}$.