MATH1040/7040 ASSIGNMENT 7 SOLUTIONS

1. $y = -6x^2 - 7$, so

$$\begin{aligned} y' &= 2 \times \left(-6x^{2-1}\right) \\ &= -12x \end{aligned}$$

2. $y = -7x + 4x^6 + \dfrac{3}{x^4}$, so $y = -7x + 4x^6 + 3x^{-4}$, so

$$\begin{aligned} y' &= 1 \times \left(-7x^{1-1}\right) + 6 \times 4x^{6-1} - 4 \times 3x^{-4-1} \\ &= -7 + 24x^5 - 12x^{-5} \\ &= -7 + 24x^5 - \frac{12}{x^5} \end{aligned}$$

3. $y = 2\sin x - 5\cos x$, so

$$\begin{aligned} y' &= 2\cos x - 5 \times (-\sin x) \\ &= 2\cos x + 5\sin x \end{aligned}$$

4. $y = -4\ln x - 7e^x + x^8$, so

$$\begin{aligned} y' &= -4 \times \frac{1}{x} - 7e^x + 8 \times x^{8-1} \\ &= -\frac{4}{x} - 7e^x + 8x^7 \end{aligned}$$

Hence $y' = -\dfrac{4}{x} - 7e^x + 8x^7$.

5. Q1 $f'(x) = -3x^2 - 18x - 24$

Q2 $f'(x) = 0$, so from Q1 , $-3x^2 - 18x - 24 = 0$, so we use $a = -3, b = -18, c = -24$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{18 \pm \sqrt{(-18)^2 - 4 \times (-3) \times (-24)}}{2 \times (-3)} \\ &= \frac{18 \pm \sqrt{324 - 288}}{-6} \\ &= \frac{18 \pm \sqrt{36}}{-6} \\ &= \frac{18+6}{-6} \ \ or \ \ \frac{18-6}{-6} \\ &= \frac{24}{-6} \ \ or \ \ \frac{12}{-6} \\ &= -4 \ \ or \ \ -2 \end{aligned}$$

Q3 $f''(x) = -6x - 18$

Q4 $f'(-4) = -3 \times (-4)^2 - 18 \times (-4) - 24 = 0$

6. Let $u = 5 - 8x$, then $u' = -8$.
Let $v = 2x^2 + 2$, then $v' = 4x$.
Product rule: $y' = u'v + uv'$.

Substitute u, u', v and v' into the product rule:

$$\begin{aligned} y' &= -8 \times \left(2x^2 + 2\right) + (5 - 8x) \times 4x \\ &= -16x^2 - 16 + 20x - 32x^2 \end{aligned}$$

Hence $y' = -48x^2 + 20x - 16$.

7. Let $u = -5x^2 + 5$, then $u' = -10x$.
Let $v = 4x^2 + 5x$, then $v' = 8x + 5$.
Quotient rule: $y' = \dfrac{u'v - uv'}{v^2}$, so

$$\begin{aligned} y' &= \frac{-10x \times \left(4x^2 + 5x\right) - \left(-5x^2 + 5\right) \times (8x + 5)}{\left(4x^2 + 5x\right)^2} \\ &= \frac{-40x^3 - 50x^2 - \left(-40x^3 - 25x^2 + 40x + 25\right)}{\left(4x^2 + 5x\right)^2} \\ &= \frac{-40x^3 - 50x^2 + 40x^3 + 25x^2 - 40x - 25}{\left(4x^2 + 5x\right)^2} \end{aligned}$$

Hence $y' = \dfrac{-25x^2 - 40x - 25}{\left(4x^2 + 5x\right)^2}$.

8. Let $u = 2z + 8$, then $u' = 2$.
Let $v = -8 - 7z$, then $v' = -7$.
Quotient rule: $y' = \dfrac{u'v - uv'}{v^2}$, so

$$y' = \frac{2 \times (-8 - 7z) - (2z + 8) \times (-7)}{\left(-8 - 7z\right)^2} = \frac{-16 - 14z + 14z + 56}{\left(-8 - 7z\right)^2} = \frac{40}{\left(-8 - 7z\right)^2}.$$

Hence $y' = \dfrac{40}{\left(-8 - 7z\right)^2}$.

9. **(a)** Let $y = x^2 - 2x + 6$. (1)

(i) $y' = 2x - 2$ (2)

(ii) Let (2) equal 8 and solve for x: $2x - 2 = 8 \rightarrow$ $2x = 10 \rightarrow$ $x = 5$.

Let (2) equal -4 and solve for x: $2x - 2 = -4 \rightarrow$ $2x = -2 \rightarrow$ $x = -1$.

When the slope is 8 the x coordinate is 5 and when the slope is -4 the x coordinate is -1.

(iii) When $x = -1$, $y = x^2 - 2x + 6 = 1 + 2 + 6 = 9$. When $x = 5$, $y = 5^2 - 10 + 6 = 21$.

(iv) Let (2) equal 0 and solve for x: $2x - 2 = 0 \rightarrow$ $2x = 2 \rightarrow$ $x = 1$.
So at $x = 1$ the slope of the track is 0 (that is, the track is level).

(b) Let $y = \frac{1}{3}x^3 + x^2 - 24x + 4$. (3)

(i) $y' = x^2 + 2x - 24 = (x + 6)(x - 4)$ (4)

Let (4) equal 0 and solve for x: $(x + 6)(x - 4) = 0 \rightarrow$ $x = -6$ or 4.

Note that the quadratic formula could also be used to find these values.

(ii) Let (4) equal 11 and solve for x: $x^2 + 2x - 24 = 11$

$\rightarrow x^2 + 2x - 35 = (x + 7)(x - 5) = 0 \rightarrow$ $x = -7$ or 5.