THE UNIVERSITY OF QUEENSLAND

AUSTRALIA

MATH1040

Basic Mathematics

Revision Guide Solutions

2nd Edition, 2011

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1.1 Thinking about maths

Questions

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1. For each question, decide which answer(s) are most likely to be correct. Explain why.

(**1**) In an Olympic 400m running race, the maximum speed attained by the winner (in metres per second) is:

i. 63.7 ii. 10 iii. 2

You might know that a fast person can run 100 metres in about 10 seconds, so that's $10m/$sec. It doesn't matter if you don't though. $2m/$sec is about walking pace (try it!), and if you could run at $63.7m/$sec that would would mean you're running the length of three cricket pitches in one second! That just leaves us with $10m/$sec

(**2**) \$1000 is invested in a bank account earning 8% interest per annum for 3 years. What is the final balance?

i. \$16728.33 ii. \$827.67 iii. \$1259.71 iv. \$1412.68

Ignoring bank fees, we can immediately rule out two answers, (i.) and (ii.). No bank is giving to give us an extra \$15,000 if we only give them \$1000. (Some shonky operators may promise this!) Answer (ii.) would mean that we have lost money, and that doesn't generally happen in Australian banks. So is it (iii.) or (iv.)? We can use rounding to help us. If we make our interest rate 10% per annum, and ignoring any compounding interest, we would have \$1300 after three years. 8% is a little less than 10%, so answer (iii.) would be a better choice than (iv.).

2. One Australian dollar (AUD) is worth 0.878 United States dollars (USD). Big Bad John spends 10 nights at a hotel in Las Vegas, at 93 USD per night. His credit card company charges a 1.5% fee to convert from USD to AUD. Roughly estimate his bill in AUD.

Big Bad John needs a rough estimate, so let's round the hotel rate to 100 USD. That brings his hotel cost to \$1000 USD. Let's round the Australian dollar to 0.9, which means

that if we add on 10% of 0.9, we get 0.99 which is close enough to 1. So if we add on 10% to John's hotel bill, that brings it to \$1100 AUD. This makes sense as the AUD is not as strong as the USD. We can ignore the 1.5% conversion fee as we only need a rough estimate.

The actual answer is \$1075.11, very close!

3. Roughly estimate the number of babies born in Australia each year.

Try: 20 million people, takes two to breed $\implies$ 10 million couples, 1.5 children/couple $\implies$ 15 million children in a lifetime of 70 years $\implies$ about 220,000 per year.

In 1998, answer was about 250,000. Close!

1.4 Absolute values

Questions

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1. Evaluate each of the following:

(**1**) $|-7.82| = 7.82$

(**2**) $-|-1| = -1$

(**3**) $|-2+5| = |3| = 3$

1.5 Simple mathematical operations

Questions

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1. Evaluate each of the following:

(**1**) $2 - (-3) = 2 + 3 = 5$

(**2**) $(-5) + 7 = 2$

(**3**) $(-6) + (-3) = -6 - 3 = -9$

(**4**) $3 \times (-4) = -12$

(**5**) $(-10) \div (-5) = 2$

1.6 Exponentiation

Questions

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1. Evaluate each of the following:

(**1**) $(-1)^{17} = -1$

(**2**) $(-1)^{356} = 1$

(**3**) $(-2)^3 = -2 \times -2 \times -2 = -8$

(**4**) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

1.7 Square roots

Questions

1. Evaluate each of the following:

(**1**) $\sqrt{9} = 3$

(**2**) $\sqrt{36} = 6$

(**3**) $\sqrt{100} = 10$

(**4**) $-\sqrt{64} = 8$

(**5**) $\sqrt{0.04} = 0.2$

(**6**) $\sqrt{0.0009} = 0.03$

1.8 Order of operations

Questions

1. To save space we have written across the page. When you are doing questions you should only have one = sign on each line.

(**1**) $12 \div 4 - 3 = 3 - 3 = 0$ and $12 \div (4 - 3) = 12 \div 1 = 12$

(**2**) $90 \div 6 \div 3 = 15 \div 3 = 5$ and $90 \div (6 \div 3) = 90 \div 2 = 45$

(**3**) $5 \times 9 - 5 = 45 - 5 = 40$ and $5 \times (9 - 5) = 5 \times 4 = 20$

(**4**) $3 \times 1 + 6 = 3 + 6 = 9$ and $3 \times (1 + 6) = 3 \times 7 = 21$

(**5**) $48 \div 3 \times 4 = 16 \times 4 = 64$ and $48 \div (3 \times 4) = 48 \div 12 = 4$

2. (**1**) $8 \div 4 \times 2 = 2 \times 2 = 4$

(**2**) $2 + 4 \times 5 \div 2 = 2 + 20 \div 2 = 2 + 10 = 12$

(**3**) $3 \times (1 + 2) - 2 \times 4 = 3 \times 3 - 8 = 9 - 8 = 1$

(**4**) $2^3 - 4 \times 2 + (4 + 1)^2 = 8 - 8 + 5^2 = 8 - 8 + 25 = 25$

(**5**) $10 - 6 + 3 \div 3 \times 2 = 10 - 6 + 1 \times 2 = 10 - 6 + 2 = 6$

1.9 Factors and prime numbers

Questions

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1. Answer each of the following questions, showing all working:

(**1**) No, since $14 = 2 \times 7$

(**2**) No, since $58 = 2 \times 29$

(**3**) No, since $15 = 3 \times 5$

(**4**) Yes, its only factors are 19 and 1

(**5**) No, since $8 = 2 \times 4$

(**6**) Yes, its only factors are 97 and 1

2. Write each of the following as the product of prime factors (if it's not already prime):

(**1**) $12 = 3 \times 4 = 3 \times 2 \times 2$

(**2**) 31 is prime

(**3**) $48 = 2 \times 24 = 2 \times 3 \times 8 = 2 \times 3 \times 2 \times 4 = 2 \times 3 \times 2 \times 2 \times 2$

1.10 Fractions: Multiplication and Division

Questions

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1. Answer each of the following questions, showing all working:

(**1**)

$$\begin{aligned}\frac{0}{-16} \times \frac{-1}{-1} &= 0 \times \frac{-1}{-1} \\ &= 0\end{aligned}$$

(**2**)

$$\begin{aligned}\frac{-8}{-7} \times \frac{14}{12} &= \frac{\cancel{2} \times 4}{\cancel{7}} \times \frac{\cancel{2} \times \cancel{7}}{\cancel{2} \times \cancel{2} \times 3} \\ &= \frac{4}{1} \times \frac{1}{3} \\ &= \frac{4 \times 1}{1 \times 3} \\ &= \frac{4}{3} \\ &= 1\frac{1}{3}\end{aligned}$$

(**3**)

$$\begin{aligned}\frac{1}{17} \times \frac{\cancel{3}}{\cancel{3}} &= \frac{1}{17} \times \frac{1}{1} \\ &= \frac{1}{17}\end{aligned}$$

(**4**)

$$\frac{-13}{11} \times \frac{-13}{16} = \frac{-13 \times (-13)}{11 \times 16}$$
$$= \frac{169}{176}$$

(**5**)

$$\frac{7}{8} \times \frac{-4}{3} = \frac{7}{\cancel{4} \times 2} \times \frac{\cancel{4} \times (-1)}{3}$$
$$= \frac{7}{2} \times \frac{-1}{3}$$
$$= \frac{7 \times (-1)}{2 \times 3}$$
$$= \frac{-7}{6}$$
$$= -1\frac{1}{6}$$

(**6**)

$$\frac{-3}{4} \div \frac{4}{10} = \frac{-3}{4} \times \frac{10}{4}$$
$$= \frac{-3}{4} \times \frac{\cancel{2} \times 5}{\cancel{2} \times 2}$$
$$= \frac{-3}{4} \times \frac{5}{2}$$
$$= \frac{-3 \times 5}{4 \times 2}$$
$$= \frac{-15}{8}$$
$$= -1\frac{7}{8}$$

(**7**)

$$\begin{aligned}\frac{-10}{11} \div \frac{16}{-6} &= \frac{-10}{11} \times \frac{-6}{16} \\ &= \frac{\cancel{2} \times 5}{11} \times \frac{\cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times 4} \\ &= \frac{5}{11} \times \frac{3}{4} \\ &= \frac{5 \times 3}{11 \times 4} \\ &= \frac{15}{44}\end{aligned}$$

(**8**)

$$\begin{aligned}\frac{7}{17} \div \frac{7}{1} &= \frac{\cancel{7}}{17} \times \frac{1}{\cancel{7}} \\ &= \frac{1}{17} \times \frac{1}{1} \\ &= \frac{1}{17}\end{aligned}$$

(**9**)

$$\begin{aligned}\frac{7}{4} \div \frac{8}{-12} &= \frac{7}{4} \times \frac{-12}{8} \\ &= \frac{7}{4} \times \frac{\cancel{4} \times (-3)}{\cancel{4} \times 2} \\ &= \frac{7}{4} \times \frac{-3}{2} \\ &= \frac{7 \times (-3)}{4 \times 2} \\ &= \frac{-21}{8} \\ &= -2\frac{5}{8}\end{aligned}$$

(**10**)

$$\begin{aligned}\frac{11}{2} \div \frac{5}{-2} &= \frac{11}{\cancel{2}} \times \frac{\cancel{-2}}{5} \\ &= \frac{11}{1} \times \frac{-1}{5} \\ &= \frac{-11}{5} \\ &= -2\frac{1}{5}\end{aligned}$$

1.10 Fractions: Addition and subtraction

Questions

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1. Answer each of the following questions, showing all working:

(**1**)

$$\begin{aligned}\frac{7}{2} + \frac{-8}{5} &= \frac{7 \times 5}{2 \times 5} - \frac{8 \times 2}{5 \times 2} \\ &= \frac{35 - 16}{10} \\ &= \frac{19}{10} \\ &= 1\frac{9}{10}\end{aligned}$$

(**2**)

$$\begin{aligned}\frac{13}{18}+\frac{6}{17} &= \frac{13\times 17}{18\times 17}+\frac{6\times 18}{17\times 18}\\ &= \frac{221+108}{306}\\ &= \frac{329}{306}\\ &= 1\frac{23}{306}\end{aligned}$$

(**3**)

$$\begin{aligned}\frac{0}{14}+\frac{-4}{-6} &= 0+\frac{4}{6}\\ &= \frac{4}{6}\\ &= \frac{\cancel{2}\times 2}{\cancel{2}\times 3}\\ &= \frac{2}{3}\end{aligned}$$

(**4**)

$$\begin{aligned}\frac{14}{8}+\frac{15}{8} &= \frac{14+15}{8}\\ &= \frac{29}{8}\\ &= 3\frac{5}{8}\end{aligned}$$

(**5**)

$$\begin{aligned}\frac{-9}{14}+\frac{-3}{13} &= \frac{-9\times 13}{14\times 13}-\frac{3\times 14}{13\times 14}\\ &= \frac{-117-42}{182}\\ &= \frac{-159}{182}\\ &= -\frac{159}{182}\end{aligned}$$

(**6**)

$$\begin{aligned}\frac{1}{3}-\frac{-3}{7} &= \frac{1\times 7}{3\times 7}+\frac{3\times 3}{7\times 3}\\ &= \frac{7+9}{21}\\ &= \frac{16}{21}\end{aligned}$$

(**7**)

$$\begin{aligned}\frac{8}{3}-\frac{2}{15} &= \frac{8\times 5}{3\times 5}-\frac{2}{15}\\ &= \frac{40-2}{15}\\ &= \frac{38}{15}\\ &= 2\frac{8}{15}\end{aligned}$$

(**8**)

$$\begin{aligned}\frac{-9}{-4}-\frac{-1}{9}&=\frac{9\times 9}{4\times 9}+\frac{1\times 4}{9\times 4}\\&=\frac{81+4}{36}\\&=\frac{85}{36}\\&=2\frac{13}{36}\end{aligned}$$

(**9**)

$$\begin{aligned}\frac{-14}{10}-\frac{12}{8}&=\frac{-14\times 4}{10\times 4}-\frac{12\times 5}{8\times 5}\\&=\frac{-56-60}{40}\\&=\frac{-116}{40}\\&=-\frac{\cancel{4}\times 29}{\cancel{4}\times 10}\\&=-\frac{29}{10}\\&=-2\frac{9}{10}\end{aligned}$$

(**10**)

$$\begin{aligned}\frac{13}{12}-\frac{-8}{20}&=\frac{13\times 5}{12\times 5}+\frac{8\times 3}{20\times 3}\\&=\frac{65+24}{60}\\&=\frac{89}{60}\\&=1\frac{29}{60}\end{aligned}$$

2. Answer each of the following questions, showing all working. Remember to apply BEDMAS.

(**1**)

$$\begin{aligned}
\frac{0}{4} \div \frac{-35}{18} + \frac{53}{-38} - \frac{-7}{-38} &= 0 \div \frac{-35}{18} + \frac{53}{-38} - \frac{7}{38} \\
&= 0 + \frac{53}{-38} - \frac{7}{38} \\
&= 0 - \frac{53}{38} - \frac{7}{38} \\
&= \frac{-53}{38} - \frac{7}{38} \\
&= \frac{-53 - 7}{38} \\
&= \frac{-60}{38} \\
&= -\frac{\cancel{2} \times 30}{\cancel{2} \times 19} \\
&= -\frac{30}{19} \\
&= -1\frac{11}{19}
\end{aligned}$$

(**2**)

$$\begin{aligned}
\frac{4}{-1}-\frac{1}{-15}+\frac{-1}{15}+\frac{-8}{13} &= \frac{-4\times 15}{1\times 15}+\frac{1}{15}+\frac{-1}{15}+\frac{-8}{13}\\
&= \frac{-60+1}{15}+\frac{-1}{15}+\frac{-8}{13}\\
&= \frac{-59}{15}+\frac{-1}{15}+\frac{-8}{13}\\
&= \frac{-59-1}{15}+\frac{-8}{13}\\
&= \frac{-60}{15}+\frac{-8}{13}\\
&= -\frac{\cancel{15}\times 4}{\cancel{15}\times 1}+\frac{-8}{13}\\
&= \frac{-4}{1}+\frac{-8}{13}\\
&= \frac{-4\times 13}{1\times 13}-\frac{8}{13}\\
&= \frac{-52-8}{13}\\
&= \frac{-60}{13}\\
&= -\frac{60}{13}\\
&= -4\frac{8}{13}
\end{aligned}$$

(**3**)

$$\begin{aligned}
\frac{-8}{6} \div \frac{56}{-29} \div \frac{58}{27} \div \frac{36}{38} &= \frac{-8}{6} \times \frac{-29}{56} \div \frac{58}{27} \div \frac{36}{38} \\
&= \frac{\cancel{2} \times \cancel{4} \times (-1)}{\cancel{2} \times 3} \times \frac{-29}{\cancel{4} \times 14} \div \frac{58}{27} \div \frac{36}{38} \\
&= \frac{-1}{3} \times \frac{-29}{14} \div \frac{58}{27} \div \frac{36}{38} \\
&= \frac{-1 \times (-29)}{3 \times 14} \div \frac{58}{27} \div \frac{36}{38} \\
&= \frac{29}{42} \div \frac{58}{27} \div \frac{36}{38} \\
&= \frac{29}{42} \times \frac{27}{58} \div \frac{36}{38} \\
&= \frac{\cancel{29}}{\cancel{3} \times 14} \times \frac{\cancel{3} \times 9}{\cancel{29} \times 2} \div \frac{36}{38} \\
&= \frac{1}{14} \times \frac{9}{2} \div \frac{36}{38} \\
&= \frac{1 \times 9}{14 \times 2} \div \frac{36}{38} \\
&= \frac{9}{28} \div \frac{36}{38} \\
&= \frac{9}{28} \times \frac{38}{36} \\
&= \frac{\cancel{9}}{28} \times \frac{\cancel{2} \times 19}{\cancel{2} \times \cancel{9} \times 2} \\
&= \frac{1}{28} \times \frac{19}{2} \\
&= \frac{1 \times 19}{28 \times 2} \\
&= \frac{19}{56}
\end{aligned}$$

(4)

$$\begin{aligned}
\frac{6}{-8} \times \frac{-45}{-12} + \frac{-60}{-16} - \frac{-36}{-48} &= \frac{\cancel{2} \times (-3)}{\cancel{2} \times 4} \times \frac{\cancel{3} \times 15}{\cancel{3} \times 4} + \frac{60}{16} - \frac{36}{48} \\
&= \frac{-3}{4} \times \frac{15}{4} + \frac{60}{16} - \frac{36}{48} \\
&= \frac{-3 \times 15}{4 \times 4} + \frac{60}{16} - \frac{36}{48} \\
&= \frac{-45}{16} + \frac{60}{16} - \frac{36}{48} \\
&= \frac{-45 + 60}{16} - \frac{36}{48} \\
&= \frac{15}{16} - \frac{36}{48} \\
&= \frac{15 \times 3}{16 \times 3} - \frac{36}{48} \\
&= \frac{45 - 36}{48} \\
&= \frac{9}{48} \\
&= \frac{\cancel{3} \times 3}{\cancel{3} \times 16} \\
&= \frac{3}{16}
\end{aligned}$$

(**5**)

$$\begin{aligned}
\left(\frac{-8}{8}-\frac{-60}{56}\right)\times\frac{-3}{6}-\frac{-18}{-14} &= \left(\frac{-8\times 7}{8\times 7}+\frac{60}{56}\right)\times\frac{-3}{6}-\frac{18}{14}\\
&= \frac{-56+60}{56}\times\frac{-3}{6}-\frac{18}{14}\\
&= \frac{4}{56}\times\frac{-3}{6}-\frac{18}{14}\\
&= \frac{\cancel{4}\times 1}{\cancel{4}\times 14}\times\frac{-3}{6}-\frac{18}{14}\\
&= \frac{1}{14}\times\frac{-3}{6}-\frac{18}{14}\\
&= \frac{1}{14}\times\frac{\cancel{3}\times(-1)}{\cancel{3}\times 2}-\frac{18}{14}\\
&= \frac{1}{14}\times\frac{-1}{2}-\frac{18}{14}\\
&= \frac{1\times(-1)}{14\times 2}-\frac{18}{14}\\
&= \frac{-1}{28}-\frac{18}{14}\\
&= \frac{-1}{28}-\frac{18\times 2}{14\times 2}\\
&= \frac{-1-36}{28}\\
&= \frac{-37}{28}\\
&= -\frac{37}{28}\\
&= -1\frac{9}{28}
\end{aligned}$$

3. There are some very common errors when dealing with fractions. Each of the following examples is **incorrect**. In each case, work out the correct answer. (Parts (b) and (c) are particularly common errors.)

(**1**) $\dfrac{1}{2}+\dfrac{3}{4}=\dfrac{1+3}{4+2}=\dfrac{4}{6}$

Thinking about this shows that the answer is wrong. We have half of something plus three-quarters of something, yet our answer is less than 1.

We don't have common denominators here so we can't add the numerators. Using a common denominator of 4, we have

$$\frac{2}{4}+\frac{3}{4}=\frac{2+3}{4}=\frac{5}{4}$$

(**2**) $\dfrac{4+2}{2}=4$

We need to simplify the numerator first, giving us $\dfrac{6}{2}=3$. You cannot cancel anything if there are numbers separated by a + or $-$ sign.

(**3**) $\dfrac{6+4}{1+4}=\dfrac{6}{1}=6$

Again, we need to simplify the numerator and denominator first, giving us $\dfrac{10}{5}=2$. You cannot cancel anything if there are numbers separated by a + or $-$ sign.

(**4**) $\dfrac{2}{5}\times 3=\dfrac{2\times 3}{5\times 3}=\dfrac{6}{15}$

We can easily turn 3 into a fraction, $\dfrac{3}{1}$. So we now have $\dfrac{2}{5}\times\dfrac{3}{1}=\dfrac{6}{5}$

(**5**) $\dfrac{2}{2} = 0.$

Anything divided by itself (except 0) is 1.
