# PART A

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| 1. When the prime numbers are written in increasing order, $x$ is the prime number that follows 7. | **11** |
| 2. $x = 5 - 4 \div 2 + 2 + 2^0 - 5 \times 6 \div 2$ | **-9** |
| 3. $x = \left| -2 - 3 \right| - \left| 2 + 3 \right|$ | **0** |
| 4. $x = \dfrac{3}{4} + \dfrac{5}{12} - \dfrac{\left| -1 \right|}{6}$ | **1** |
| 5. $y = 7x - 2$ where $y = -16$ | **-2** |
| 6. $3x - 5 = 1 - 2x$ | **6/5** |
| 7. $x = \dfrac{\left( f - 5 \right)^2}{2} - \sqrt{f}$ where $f = 9$ | **5** |
| 8. $\left( \sqrt{1 - 6x} \right)^2 = 7$ | **-1** |
| 9. $\dfrac{1}{x} + 4 = 0$ | **-1/4** |
| 10. $\left( 2\sqrt{5} \right)^2 \div 4x = \dfrac{1}{2}$ | **10** |
| 11. $\sqrt{28} = -x\sqrt{7}$ | **-2** |
| 12. $4^x = 1$ | **0** |
| 13. $\dfrac{5^x}{5^{-1}} = 5^2$ | **1** |
| 14. $3 = 27^{1/x}$ | **3** |

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| 15. $\displaystyle\sum_{i=-2}^{2}\left(i^2-1\right)=x$ | **5** |
| 16. $\displaystyle\sum_{i=1}^{3}\left(i^2x-i\right)=8$ | **1** |
| 17. $\displaystyle\sum_{i=0}^{1}2x=8$ | **4** |
| 18. $\dfrac{2}{x+1}=\dfrac{3}{2x-2}$ | **7** |
| 19. $\dfrac{x^2-6x+9}{x-3}=2x+6$ | **-9** |
| 20. If $A=\{1,2,3,4\}$ and $B=\{1,4,5\}$ then $B\setminus A=\{x\}$ | **5** |
| 21. The probability of any event is a value between $x+1$ and $x+2$ (inclusive). | **-1** |
| 22. When a die is rolled, the probability that the number showing is 3 given that the number is odd, is $1/x$. | **3** |
| 23. The point $(4, 2)$ is a distance of $x\sqrt{5}$ from the origin. | **2** |
| 24. $x$ is the $y$-intercept of $y(t)=4t-2$ | **-2** |
| 25. $x$ is the gradient of a line perpendicular to $y(t)=-\dfrac{1}{2}t+5$ | **2** |

# PART B

**1.** A bus can hold 36 passengers. If 1128 conference attendees are being bussed to their conference dinner location, how many buses are needed?

**Solution**

1128/36 = 31.333. They need **32 buses.**

**2.** Evaluate $\dfrac{1}{4} - \dfrac{4}{3} \times \left(\dfrac{1}{4}\right)^3 \div \left(\dfrac{1}{2}\right)^2$, leaving the answer as a fraction. Show all working.

**Solution**

$$\frac{1}{4} - \frac{4}{3} \times \left(\frac{1}{4}\right)^3 \div \left(\frac{1}{2}\right)^2 = \frac{1}{4} - \frac{4}{3} \times \frac{1}{4^3} \times \frac{2^2}{1} = \frac{1}{4} - \frac{4^2}{3 \times 4^3} = \frac{1}{4} - \frac{1}{3 \times 4} = \frac{3-1}{12} = \frac{1}{6}$$

**3.** Solve $-\left|3x + 4\right| = -2$. Show all working.

**Solution**

$$-\left|3x+4\right| = -2 \iff \begin{cases} -3x - 4 = -2 \\ -3x - 4 = 2 \end{cases} \iff \begin{cases} 3x = -2 \\ 3x = -6 \end{cases} \iff \begin{cases} x = -\dfrac{2}{3} \\ x = -2 \end{cases}$$

**4. (a)** Find all $x$ for which $-5 + 2(x+1) \geq 3(x+2)$, writing your answer in equality form (for instance $x < ...$). Show all working.

**Solution**

$$-5 + 2(x+1) \geq 3(x+2)$$

$$2x - 3 \geq 3x + 6 \iff -x \geq 9 \iff x \leq -9$$

**(b)** Write your answer to part **(a)** in interval format.

**Solution**

$$x \in \left(-\infty, -9\right]$$

**5.** Expand $\left(\sqrt{x} - 7\right)\left(\sqrt{x} + 3\right)$. Show all working.

**Solution**

$$\left(\sqrt{x} - 7\right)\left(\sqrt{x} + 3\right) = \sqrt{x}\sqrt{x} + 3\sqrt{x} - 7\sqrt{x} - 21 = x - 4\sqrt{x} - 21$$

**6.** Simplify $\dfrac{\sqrt{27}\left(\sqrt{2} - 2\right)\left(\sqrt{18} + 3\right)}{\sqrt{3}}$. Show all working.

**Solution**

$$\frac{\sqrt{27}\left(\sqrt{2} - 2\right)\left(\sqrt{18} + 3\right)}{\sqrt{3}} = \frac{3\sqrt{3}\left(\sqrt{2} - 2\right)\left(\sqrt{18} + 3\right)}{\sqrt{3}} =$$

$$3\left(\sqrt{2} - 2\right)\left(\sqrt{18} + 3\right) = 3\left(\sqrt{36} + 3\sqrt{2} - 2\sqrt{18} - 6\right) = 3\left(6 + 3\sqrt{2} - 6\sqrt{2} - 6\right) = 3\left(-3\sqrt{2}\right) = -9\sqrt{2}$$

**7.** Simplify $\dfrac{(ab)^3 \times a^{-2}b \div \left(-a^2 b^{-1/3}\right)^{-3}}{b^{-3}}$. Show all working.

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**Solution**

$$\frac{(ab)^3 \times a^{-2}b \div \left(-a^2 b^{-1/3}\right)^{-3}}{b^{-3}} = \frac{(ab)^3 \times a^{-2}b \div \left(-a^{-6}b\right)}{b^{-3}} = \frac{(ab)^3 \times a^{-2}b \times \left(-a^6 b^{-1}\right)}{b^{-3}} =$$

$$-\frac{(ab)^3 \times a^4}{b^{-3}} = -\frac{a^3 \times b^3 \times a^4}{b^{-3}} = -a^7 b^6$$

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**8.** Write in summation (sigma) notation:
$$j + 2j^2 + 3j^3 + 4j^4 + 5j^5 + 6j^6$$

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**Solution**

$$j + 2j^2 + 3j^3 + 4j^4 + 5j^5 + 6j^6 = \sum_{i=1}^{6} ij^i$$

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**9.** Write in summation (sigma) notation.
$$\left(-x^3 - 1\right) + \left(x^5 - 2\right) + \left(-x^7 - 3\right)$$

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**Solution**

$$\left(-x^3 - 1\right) + \left(x^5 - 2\right) + \left(-x^7 - 3\right) = \sum_{i=1}^{3} \left((-1)^i x^{2i+1} - i\right)$$

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**10.** Solve $10x^{10} - 7x - 10 = \left(5x^5 + 5\right)\left(2x^5 - 2\right) + 2$. Show all working.

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**Solution**

$$10x^{10} - 7x - 10 = \left(5x^5 + 5\right)\left(2x^5 - 2\right) + 2$$

$$10x^{10} - 7x - 10 = 10x^{10} - 10 + 2$$

$$x = -\frac{2}{7}$$

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**11.** A survey revealed that 50 people liked coffee and 40 people liked tea. Both these figures include 35 who liked both coffee and tea. Finally 10 did not like either coffee or tea. How many people responded to the survey? Show all working.

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**Solution**

A people who like coffee        A = { 50 }
B people who like tea           B = { 40 }
A ∩ B = 35 like coffee and tea
B \ (A ∩ B ) = 5 only like tea
A \ (A ∩ B ) = 15 only like coffee
35 + 5 + 15 + 10 = 65 people

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**12.** A card is selected from a well-shuffled deck of 52 cards.
**(a)** What is the probability that it is an ace **and** a heart?

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**Solution**

Sample space is 52 cards
P(ace) = 4 /52 = 1/ 13

P(heart) = 13 / 52
P(ace and heart) = 1/ 52

**(b)** What is the probability that it is an ace **or** a heart?

**Solution**
P(ace or heart) = 1/13 + 13/52 − 1/52 = 16/52 = 4/13

13. Let $A = \{-2,0,2\}$, $B = \{0,2,4\}$ and $C = \{0,1,5\}$
    **(a)** Write down the set $A \backslash (B \cup C)$.
    **(b)** Write down the set $(A \cap B) \backslash (A \cap C)$.
    **(c)** Write down the set $(A \cup B) \cap C$.
    **(d)** Write down the set $((C \backslash B) \backslash A) \backslash C$.
    **(e)** Write down the set $(C \cup \varnothing) \cap (A \cup \varnothing) \cap (B \cup \varnothing)$.

**Solution**
(a) $A \backslash (B \cup C)$ = A \ {0, 2, 4, 1, 5} = {-2}
(b) $(A \cap B) \backslash (A \cap C)$ = {0, 2}\ {0} = {2)
(c) $(A \cup B) \cap C$ = {-2, 0, 2, 4} inter C= {0}
(d) $((C \backslash B) \backslash A) \backslash C$ = {{1, 5} \ {-2, 0, 2} )\ C = {1, 5}\ {0, 1, 5}= {}
(e) $(C \cup \varnothing) \cap (A \cup \varnothing) \cap (B \cup \varnothing)$ = {0, 1, 5} inter {-2, 0, 2} inter {0, 2, 4}={0 }

14. Let L be the line $y = -x + 2$ and let M be the line perpendicular to L passing through the point (-2, 4).
    **(a)** Find the equation of the line M.

**Solution**
L : $y = -x + 2$
M : $y = x + c$    passes through $(-2,4) \implies 4 = -2 + c \implies c = 6$
M : $y = x + 6$

**(b)** Find the distance between the *y*-intercept of L and the *x*-intercept of M. Express your answer as a surd in simplest form

**Solution**
$y$ - intercept of L : $x = 0 \implies y = -0 + 2 \implies x = 2$
$x$ - intercept of M : $y = 0 \implies 0 = x + 6 \implies x = -6$
Distance between $(0,2)$ and $(-6,0)$: $d = \sqrt{(0+6)^2 + (2-0)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

15. A bag contains 2 red balls, 3 green balls and 5 black balls. If three balls are drawn with replacement (*eg* a ball is selected at random, then replaced into the bag, then another ball is selected):
    **(a)** What is the probability that the first ball is black?
    **(b)** What is the probability of getting three green balls?
    **(c)** What is the probability of getting three balls of the same colour?
    **(d)** What is the probability that the second ball is red, given that the first ball was black?

**Solution**
i.    $P(\text{black ball}) = \dfrac{5}{10} = 0.5$

ii.   $P(3 \text{ green balls}) = \dfrac{3}{10} \times \dfrac{3}{10} \times \dfrac{3}{10} = \dfrac{27}{10^3} = 0.027$

$P(\text{3 balls same colour}) = P(\text{3 black}) + P(\text{3 red}) + P(\text{3 green}) =$

iii. $\left(\dfrac{5}{10} \times \dfrac{5}{10} \times \dfrac{5}{10}\right) + \left(\dfrac{2}{10} \times \dfrac{2}{10} \times \dfrac{2}{10}\right) + \left(\dfrac{3}{10} \times \dfrac{3}{10} \times \dfrac{3}{10}\right) =$

$\dfrac{125 + 8 + 27}{10^3} = \dfrac{160}{10^3} = 0.160$

iv. $P(\text{2nd ball is red/1st ball is black}) = \dfrac{1}{5}$   (independent event)

16. Computel and Optek are two imaginary companies that sell consumer desktop computers. The executives of Computel have launched a series of ads against rival company Optek. One of the ads shows how the price-to-performance ratio of computers sold by Computel is higher than Optek's. There is something wrong with this ad: should Computel promote their computers this way? Explain in the space below why.



Note: price-to-performance ratio refers to the price and performance of the computer.

**Solution**
The highest ratio means that either Computel computers are higher priced than Optek's, or their performance is worse or both. Computel is advertising high price, low performance computers!