1. **(1)** $f(y) = -2y^2 - 9y - 1$, so
$f(-4) = -2 \times (-4)^2 - 9 \times (-4) - 1 = -32 + 36 - 1 = 3$

(2) $y(-8y + 7) = 0$, so

$$y = 0 \qquad \textit{or} \qquad \begin{aligned} -8y + 7 &= 0 \\ -8y &= -7 \\ y &= \frac{7}{8} \end{aligned}$$

(3) $-3z^2 - 6z - 6 = 0$, so we use $a = -3, b = -6, c = -6$ in the quadratic formula. Hence

$$\begin{aligned} z &= \frac{6 \pm \sqrt{(-6)^2 - 4 \times (-3) \times (-6)}}{2 \times (-3)} \\ &= \frac{6 \pm \sqrt{36 - 72}}{-6} \\ &= \frac{6 \pm \sqrt{-36}}{-6} \end{aligned}$$

Hence there is no solution.

(4) To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

i. $-3y(-4 - 6y) = 0$, so

$$\begin{aligned} -3y &= 0 \\ y &= 0 \end{aligned} \qquad \textit{or} \qquad \begin{aligned} -4 - 6y &= 0 \\ -6y &= 4 \\ y &= \frac{4}{-6} \\ y &= -\frac{2}{3} \end{aligned}$$

ii. $(1 - 2z)(9z + 10) = 0$, so

$$\begin{aligned} 1 - 2z &= 0 \\ -2z &= -1 \\ z &= \frac{1}{2} \end{aligned} \qquad \textit{or} \qquad \begin{aligned} 9z + 10 &= 0 \\ 9z &= -10 \\ z &= -\frac{10}{9} \end{aligned}$$

iii. $6(-3z - 7)(-3z + 1) = 0$, so

$$\begin{aligned} -3z - 7 &= 0 \\ -3z &= 7 \\ z &= -\frac{7}{3} \end{aligned} \qquad \textit{or} \qquad \begin{aligned} -3z + 1 &= 0 \\ -3z &= -1 \\ z &= \frac{1}{3} \end{aligned}$$

iv. $(8 - 8x)^3 = 0$, so $8 - 8x = 0$, so $-8x = -8$, so $x = \frac{-8}{-8}$, so $x = 1$

(5) $f(x) = -7 + \left|x^2\right|$
When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;
- we can square any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of x can be substituted into f .

(6) $f(w) = 3 + |\sqrt{w}|$

When evaluating the range, we need to keep in mind the following (starting with variable w):

- square root is always positive or 0, so $\sqrt{w} \geq 0$;
- absolute value is always positive or 0, so $|\sqrt{w}| \geq 0$;
- so $3 + |\sqrt{w}| \geq 3$.

Hence, the range of this function is $[3, \infty)$.

(7) $f(z) = \dfrac{6}{|z| + 10}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $|z| + 10 \neq 0$;
- so $|z| \neq -10$;
- we can find the absolute value of any number. It will always be positive or 0.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of z can be substituted into f .

(8) $f(x) = \left|\dfrac{-2}{-x}\right|$

When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;
- denominator of a fraction cannot be 0, so $-x \neq 0$.

Hence, the domain of this function is $(-\infty, 0) \cup (0, \infty)$, i.e. $x \neq 0$.

When evaluating the range, we need to keep in mind the following (starting with variable x):

- negative numerator usually reverse the inequality, and also this fraction can't be 0, so $\dfrac{-2}{-x} \neq 0$;
- absolute value is always positive or 0, so $\left|\dfrac{-2}{-x}\right| > 0$.

Hence, the range of this function is $(0, \infty)$.

(9) **

$f(x) = \dfrac{-10}{10 + \sqrt{x}}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- square root is always positive or 0, so $0 \leq \sqrt{x}$;
- fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
- so $10 \leq 10 + \sqrt{x}$.

Hence, the range of this function is $[-1, 0)$.

2. **(1)** $f(y) = -3y^2 - 10y - 10$, so

$f(5) = -3 \times 5^2 - 10 \times 5 - 10 = -75 - 50 - 10 = -135$

(2) $(7z - 4)(-10z + 1) = 0$, so

$$7z - 4 = 0 \qquad \textit{or} \qquad -10z + 1 = 0$$
$$7z = 4 \qquad\qquad -10z = -1$$
$$z = \frac{4}{7} \qquad\qquad z = \frac{1}{10}$$

(**3**) $-5y^2 - 10y + 15 = 0$, so we use $a = -5, b = -10, c = 15$ in the quadratic formula. Hence

$$\begin{aligned} y &= \frac{10 \pm \sqrt{(-10)^2 - 4 \times (-5) \times 15}}{2 \times (-5)} \\ &= \frac{10 \pm \sqrt{100 - (-300)}}{-10} \\ &= \frac{10 \pm \sqrt{400}}{-10} \\ &= \frac{10 + 20}{-10} \ \ or \ \ \frac{10 - 20}{-10} \\ &= \frac{30}{-10} \ \ or \ \ \frac{-10}{-10} \\ &= -3 \ \ or \ \ 1 \end{aligned}$$

(**4**) To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

i. $y\,(8 + 4y) = 0$, so

$$y = 0 \qquad or \qquad \begin{aligned} 8 + 4y &= 0 \\ 4y &= -8 \\ y &= \frac{-8}{4} \\ y &= -2 \end{aligned}$$

ii. $(-8 + 2z)\,(1 + 9z) = 0$, so

$$\begin{aligned} -8 + 2z &= 0 \\ 2z &= 8 \\ z &= \frac{8}{2} \\ z &= 4 \end{aligned} \qquad or \qquad \begin{aligned} 1 + 9z &= 0 \\ 9z &= -1 \\ z &= -\frac{1}{9} \end{aligned}$$

iii. $5\,(4z - 8)\,(-5z + 7) = 0$, so

$$\begin{aligned} 4z - 8 &= 0 \\ 4z &= 8 \\ z &= \frac{8}{4} \\ z &= 2 \end{aligned} \qquad or \qquad \begin{aligned} -5z + 7 &= 0 \\ -5z &= -7 \\ z &= \frac{7}{5} \end{aligned}$$

iv. $(-9 + 5x)^4 = 0$, so $-9 + 5x = 0$, so $5x = 9$, so $x = \dfrac{9}{5}$

(**5**) $f(w) = \sqrt{\left(\dfrac{7}{w}\right)^2}$

When determining the domain of this function, we need to keep in mind the following:

- we can only take the square root of positive numbers or 0, so $\left(\dfrac{7}{w}\right)^2 \geq 0$;
- we can square any number;
- denominator of a fraction cannot be 0, so $w \neq 0$.

Hence, the domain of this function is $(-\infty, 0) \cup (0, \infty)$, i.e. $w \neq 0$.

(6) $f(w) = \sqrt{6 \times \dfrac{7}{w}}$

When evaluating the range, we need to keep in mind the following (starting with variable w):

- fraction can be 0 only if numerator is 0, so $\dfrac{7}{w} \neq 0$;
- square root is always positive or 0, so $\sqrt{6 \times \dfrac{7}{w}} > 0$.

Hence, the range of this function is $(0, \infty)$.

(7) $f(z) = \dfrac{12}{-9 + \sqrt{z}}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $-9 + \sqrt{z} \neq 0$;
- so $\sqrt{z} \neq 9$;
- we can only take the square root of positive number or 0, so $z \neq 81$ and $0 \leq z$.

Hence, the domain of this function is $[0, 81) \cup (81, \infty)$, i.e. $z \neq 81$ and $0 \leq z$.

(8) $f(w) = \sqrt{w^2} + 5$

When determining the domain of this function, we need to keep in mind the following:

- we can only take the square root of positive numbers or 0, so $w^2 \geq 0$;
- we can square any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of w can be substituted into f .

When evaluating the range, we need to keep in mind the following (starting with variable w):

- squaring always gives a positive or 0, so $w^2 \geq 0$;
- square root is always positive or 0, so $\sqrt{w^2} \geq 0$;
- so $\sqrt{w^2} + 5 \geq 5$.

Hence, the range of this function is $[5, \infty)$.

(9) **

$f(z) = \dfrac{1}{z^2 + 3}$

When evaluating the range, we need to keep in mind the following (starting with variable z):

- squaring always gives a positive or 0, so $0 \leq z^2$;
- fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
- so $3 \leq z^2 + 3$.

Hence, the range of this function is $(0, \frac{1}{3}]$.

3. **(1)** $f(z) = -7z - 6$, so
$f(9) = -7 \times 9 - 6 = -63 - 6 = -69$

(2) $7y(3y - 3) = 0$, so

$$7y = 0 \qquad \textit{or} \qquad 3y - 3 = 0$$
$$y = 0 \qquad\qquad 3y = 3$$
$$y = \frac{3}{3}$$
$$y = 1$$

(3) $5z^2 - 15z - 50 = 0$, so we use $a = 5, b = -15, c = -50$ in the quadratic formula. Hence

$$\begin{aligned} z &= \frac{15 \pm \sqrt{(-15)^2 - 4 \times 5 \times (-50)}}{2 \times 5} \\ &= \frac{15 \pm \sqrt{225 - (-1000)}}{10} \\ &= \frac{15 \pm \sqrt{1225}}{10} \\ &= \frac{15 + 35}{10} \;\; or \;\; \frac{15 - 35}{10} \\ &= \frac{50}{10} \;\; or \;\; \frac{-20}{10} \\ &= 5 \;\; or \;\; -2 \end{aligned}$$

(4) To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

i. $10z\,(8 + 3z) = 0$, so

$$\begin{aligned} 10z &= 0 & \quad or \quad & & 8 + 3z &= 0 \\ z &= 0 & & & 3z &= -8 \\ & & & & z &= -\frac{8}{3} \end{aligned}$$

ii. $(-10 - 10x)\,(2x - 5) = 0$, so

$$\begin{aligned} -10 - 10x &= 0 & \quad or \quad & & 2x - 5 &= 0 \\ -10x &= 10 & & & 2x &= 5 \\ x &= \frac{10}{-10} & & & x &= \frac{5}{2} \\ x &= -1 \end{aligned}$$

iii. $4\,(-6 - 6y)\,(-6 + 9y) = 0$, so

$$\begin{aligned} -6 - 6y &= 0 & \quad or \quad & & -6 + 9y &= 0 \\ -6y &= 6 & & & 9y &= 6 \\ y &= \frac{6}{-6} & & & y &= \frac{6}{9} \\ y &= -1 & & & y &= \frac{2}{3} \end{aligned}$$

iv. $(3 + 7x)^9 = 0$, so $3 + 7x = 0$, so $7x = -3$, so $x = -\frac{3}{7}$

(5) $f(z) = \dfrac{-9}{\sqrt{-4 + z}}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $\sqrt{-4 + z} \neq 0$;
- we can only take the square root of positive numbers or 0, so $-4 + z > 0$;
- so $z > 4$.

Hence, the domain of this function is $(4, \infty)$, i.e. $z > 4$.

(6) $f(x) = \sqrt{2\,|x|}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- absolute value is always positive or 0, so $|x| \geq 0$;

- square root is always positive or 0, so $\sqrt{2|x|} \geq 0$.

Hence, the range of this function is $[0, \infty)$.

(7) $f(z) = \dfrac{6}{1 - 12z}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $1 - 12z \neq 0$;
- so $-12z \neq -1$;
- so $z \neq \dfrac{1}{12}$.

Hence, the domain of this function is $(-\infty, \frac{1}{12}) \cup (\frac{1}{12}, \infty)$, i.e. $z \neq \frac{1}{12}$.

(8) $f(x) = \left|x^2\right|$

When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;
- we can square any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of x can be substituted into f .

When evaluating the range, we need to keep in mind the following (starting with variable x):

- squaring always gives a positive or 0, so $x^2 \geq 0$;
- absolute value is always positive or 0, so $\left|x^2\right| \geq 0$.

Hence, the range of this function is $[0, \infty)$.

(9) **

$f(x) = \dfrac{11}{-5 + |x|}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- absolute value is always positive or 0, so $0 \leq |x|$;
- fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
- so $-5 \leq -5 + |x|$ and $-5 + |x| \neq 0$.

Hence, the range of this function is $(-\infty, -\frac{11}{5}) \cup (0, \infty)$.

4. **(1)** $f(x) = -3x^2 - 8x$, so
$f(0) = -3 \times 0^2 - 8 \times 0 = 0 + 0 = 0$

(2) $-9y(-10 + 6y) = 0$, so

$$-9y = 0 \quad \text{or} \quad -10 + 6y = 0$$
$$y = 0 \qquad\qquad 6y = 10$$
$$y = \frac{10}{6}$$
$$y = \frac{5}{3}$$

(**3**) $-4x^2 - 36x - 80 = 0$, so we use $a = -4, b = -36, c = -80$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{36 \pm \sqrt{(-36)^2 - 4 \times (-4) \times (-80)}}{2 \times (-4)} \\ &= \frac{36 \pm \sqrt{1296 - 1280}}{-8} \\ &= \frac{36 \pm \sqrt{16}}{-8} \\ &= \frac{36+4}{-8} \quad or \quad \frac{36-4}{-8} \\ &= \frac{40}{-8} \quad or \quad \frac{32}{-8} \\ &= -5 \quad or \quad -4 \end{aligned}$$

(**4**) To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

i. $9x(-3x - 4) = 0$, so

$$\begin{aligned} 9x &= 0 & \quad or \quad & & -3x - 4 &= 0 \\ x &= 0 & & & -3x &= 4 \\ & & & & x &= -\frac{4}{3} \end{aligned}$$

ii. $(-3x + 7)(-4 + 8x) = 0$, so

$$\begin{aligned} -3x + 7 &= 0 & \quad or \quad & & -4 + 8x &= 0 \\ -3x &= -7 & & & 8x &= 4 \\ x &= \frac{7}{3} & & & x &= \frac{4}{8} \\ & & & & x &= \frac{1}{2} \end{aligned}$$

iii. $6(-10x - 1)(-8x - 8) = 0$, so

$$\begin{aligned} -10x - 1 &= 0 & \quad or \quad & & -8x - 8 &= 0 \\ -10x &= 1 & & & -8x &= 8 \\ x &= -\frac{1}{10} & & & x &= \frac{8}{-8} \\ & & & & x &= -1 \end{aligned}$$

iv. $(10z - 1)^1 = 0$, so $10z - 1 = 0$, so $10z = 1$, so $z = \frac{1}{10}$

(**5**) $f(w) = -3\sqrt{w - 4}$
When determining the domain of this function, we need to keep in mind the following:

- we can only take the square root of positive numbers or 0, so $w - 4 \geq 0$;
- so $w \geq 4$.

Hence, the domain of this function is $[4, \infty)$, i.e. $w \geq 4$.

(**6**) $f(w) = 2 + \sqrt{w^2}$
When evaluating the range, we need to keep in mind the following (starting with variable w):

- squaring always gives a positive or 0, so $w^2 \geq 0$;
- square root is always positive or 0, so $\sqrt{w^2} \geq 0$;

- so $2+\sqrt{w^2}\geq 2$.

Hence, the range of this function is $[2,\infty)$.

(7) $f(z)=\dfrac{-12}{z^2+4}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $z^2+4\neq 0$;
- so $z^2\neq -4$;
- we can square any number and result will always be a positive number or 0.

Hence, the domain of this function is $(-\infty,\infty)$, i.e. any value of z can be substituted into f .

(8) $f(z)=-9+\dfrac{10}{z^2}$

When determining the domain of this function, we need to keep in mind the following:

- there are no square roots or absolute value signs;
- denominator of a fraction cannot be 0, so $z^2\neq 0$;
- we can square any number.

Hence, the domain of this function is $(-\infty,0)\cup(0,\infty)$, i.e. $z\neq 0$.

When evaluating the range, we need to keep in mind the following (starting with variable z):

- there are no square roots or absolute value signs;
- squaring always gives a positive or 0, so $z^2\geq 0$;
- fraction can be 0 only if numerator is 0, so $\dfrac{10}{z^2}>0$;
- so $-9+\dfrac{10}{z^2}>-9$.

Hence, the range of this function is $(-9,\infty)$.

(9) **

$f(x)=\dfrac{-3}{11x+5}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- there are no squares, square roots or absolute value signs ;
- fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 .

Hence, the range of this function is $(-\infty,0)\cup(0,\infty)$.

5. **(1)** $f(y)=6y^2+7y+8$, so

$f(4)=6\times 4^2+7\times 4+8=96+28+8=132$

(2) $9(-3-5x)(5x+8)=0$, so

$$\begin{aligned} -3-5x&=0 &\quad \textit{or} \quad&& 5x+8&=0\\ -5x&=3 &&& 5x&=-8\\ x&=-\frac{3}{5} &&& x&=-\frac{8}{5} \end{aligned}$$

(3) $y^2-10y+25=0$, so we use $a=1,b=-10,c=25$ in the quadratic formula. Hence

$$\begin{aligned} y&=\frac{10\pm\sqrt{(-10)^2-4\times 1\times 25}}{2\times 1}\\ &=\frac{10\pm\sqrt{100-100}}{2}\\ &=\frac{10\pm\sqrt{0}}{2}\\ &=\frac{10}{2}\\ &=5 \end{aligned}$$

(4) To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

i. $5z\,(9+3z) = 0$, so

$$\begin{aligned} 5z &= 0 \qquad\qquad \textit{or} \qquad\qquad & 9+3z &= 0 \\ z &= 0 & 3z &= -9 \\ & & z &= \frac{-9}{3} \\ & & z &= -3 \end{aligned}$$

ii. $(-6+2y)\,(-2+4y) = 0$, so

$$\begin{aligned} -6+2y &= 0 \qquad\qquad \textit{or} \qquad\qquad & -2+4y &= 0 \\ 2y &= 6 & 4y &= 2 \\ y &= \frac{6}{2} & y &= \frac{2}{4} \\ y &= 3 & y &= \frac{1}{2} \end{aligned}$$

iii. $5\,(-10-9x)\,(x-8) = 0$, so

$$\begin{aligned} -10-9x &= 0 \qquad\qquad \textit{or} \qquad\qquad & x-8 &= 0 \\ -9x &= 10 & x &= 8 \\ x &= -\frac{10}{9} \end{aligned}$$

iv. $(4z+1)^6 = 0$, so $4z+1 = 0$, so $4z = -1$, so $z = -\frac{1}{4}$

(5) $f(z) = \left|\frac{-1}{\sqrt{z}}\right|$

When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;
- denominator of a fraction cannot be 0, so $\sqrt{z} \neq 0$;
- we can only take the square root of positive numbers or 0, so $z > 0$.

Hence, the domain of this function is $(0, \infty)$, i.e. $z > 0$.

(6) $f(z) = \left|z^2\right| + 3$

When evaluating the range, we need to keep in mind the following (starting with variable z):

- squaring always gives a positive or 0, so $z^2 \geq 0$;
- absolute value is always positive or 0, so $\left|z^2\right| \geq 0$;
- so $\left|z^2\right| + 3 \geq 3$.

Hence, the range of this function is $[3, \infty)$.

(7) $f(x) = \frac{6}{2+x^2}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $2 + x^2 \neq 0$;
- so $x^2 \neq -2$;
- we can square any number and result will always be a positive number or 0.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of x can be substituted into f .

(8) $f(w) = -3 + |\sqrt{w}|$

When determining the domain of this function, we need to keep in mind the following:

- we can find the absolute value of any number;

- we can only take the square root of positive numbers or 0, so $w \geq 0$.

Hence, the domain of this function is $[0, \infty)$, i.e. $w \geq 0$.

When evaluating the range, we need to keep in mind the following (starting with variable w):

- square root is always positive or 0, so $\sqrt{w} \geq 0$;
- absolute value is always positive or 0, so $|\sqrt{w}| \geq 0$;
- so $-3 + |\sqrt{w}| \geq -3$.

Hence, the range of this function is $[-3, \infty)$.

(9) **

$f(x) = \dfrac{4}{1 + |x|}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- absolute value is always positive or 0, so $0 \leq |x|$;
- fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 ;
- so $1 \leq 1 + |x|$.

Hence, the range of this function is $(0, 4]$.