MATH1040/7040 SIMULTANEOUS EQUATIONS SOLUTIONS

1. (**1**) First we number the equations for convenience:

$$-3y + 5x = -2 \qquad (1)$$
$$9y - 10x = 1 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 3 , giving

$$-9y + 15x = -6 \qquad (3)$$
$$9y - 10x = 1 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-9y + 9y + 15x - 10x = -6 + 1 \qquad (5)$$

Simplifying equation (5) gives

$$5x = -5 \qquad (6)$$
$$x = -1 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$-3y + 5 \times (-1) = -2$$
$$-3y = 3 \qquad \text{so}$$
$$y = -1$$

Hence the simultaneous solution to equations (1) and (2) is $(-1, -1)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $-3 \times (-1) + 5 \times (-1) = -2$
$3 - 5 = -2$
$-2 = -2$

(2) $9 \times (-1) - 10 \times (-1) = 1$
$-9 + 10 = 1$
$1 = 1$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) First we number the equations for convenience:

$$6x = -42y + 60 \qquad (1)$$
$$-5y + 350 = -10x \qquad (2)$$

We solve these using substitution. Dividing both sides of equation (1) by 6 gives

$$x = -7y + 10 \qquad (3)$$

Substituting for x in equation (2),

$$-5y + 350 = -10 \times (-7y + 10) \qquad (4)$$

Now (4) is an equation only involving y which gives:

$$-5y + 350 = 70y - 100$$
$$-75y = -450$$
$$y = 6$$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = -7 \times 6 + 10 = -32$$

Hence the simultaneous solution to equations (1) and (2) is $(-32, 6)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ \ 6 \times (-32) &= -42 \times 6 + 60 \\ -192 &= -252 + 60 \\ -192 &= -192 \end{aligned} \qquad \begin{aligned} (2)\ \ -5 \times 6 + 350 &= -10 \times (-32) \\ -30 + 350 &= 320 \\ 320 &= 320 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 10y - 3x &= 120 \qquad (1) \\ 2y + 10x &= -82 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (2) by -5 , giving

$$\begin{aligned} 10y - 3x &= 120 \qquad (3) \\ -10y - 50x &= 410 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-10y + 10y - 50x - 3x = 410 + 120 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -53x &= 530 \qquad (6) \\ x &= -10 \qquad (7) \end{aligned}$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$\begin{aligned} 10y - 3 \times (-10) &= 120 \\ 10y &= 90 \qquad \text{so} \\ y &= 9 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(-10, 9)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ \ 10 \times 9 - 3 \times (-10) &= 120 \\ 90 + 30 &= 120 \\ 120 &= 120 \end{aligned} \qquad \begin{aligned} (2)\ \ 2 \times 9 + 10 \times (-10) &= -82 \\ 18 - 100 &= -82 \\ -82 &= -82 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**4**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 66 + 3x &= 2y \qquad (1) \\ -6x &= -12y + 60 \qquad (2) \end{aligned}$$

We solve these using substitution. Dividing both sides of equation (2) by -6 gives

$$x = 2y - 10 \qquad (3)$$

Substituting for x in equation (1),

$$66 + 3 \times (2y - 10) = 2y \qquad (4)$$

Now (4) is an equation only involving y which gives:

$$\begin{aligned} 66 + 6y - 30 &= 2y \\ 4y &= -36 \\ y &= -9 \end{aligned}$$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = 2 \times (-9) - 10 = -28$$

Hence the simultaneous solution to equations (1) and (2) is $(-28, -9)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $66 + 3 \times (-28) = 2 \times (-9)$
$66 - 84 = -18$
$-18 = -18$

(2) $-6 \times (-28) = -12 \times (-9) + 60$
$168 = 108 + 60$
$168 = 168$

Both equations turned into true statements, as required. Hence the answer is correct.)

2. (**1**) First we number the equations for convenience:

$$\begin{aligned} -5y - 8x &= -74 \qquad (1) \\ 25y + 40x &= 389 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by 5 , giving

$$\begin{aligned} -25y - 40x &= -370 \qquad (3) \\ 25y + 40x &= 389 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-25y + 25y - 40x + 40x = -370 + 389 \qquad (5)$$

Simplifying equation (5) gives

$$0 = 19 \qquad (6)$$

Statement (6) is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.

(**2**) First we number the equations for convenience:

$$\begin{aligned} -2x &= 9y - 5 \qquad (1) \\ 0 &= 10x + 20 + 90y \qquad (2) \end{aligned}$$

We solve these using substitution. Rearranging equation (2) with x on the left-hand side gives

$$-10x = 90y + 20 \qquad (3)$$

Dividing both sides of (3) by -10, gives

$$x = -9y - 2 \qquad (4)$$

Substituting for x in equation (1),

$$-2 \times (-9y - 2) = 9y - 5 \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$\begin{aligned} 18y + 4 &= 9y - 5 \\ 9y &= -9 \\ y &= -1 \end{aligned}$$

Next we substitute the value for y into equation (4) to obtain the value for x, giving

$$x = -9 \times (-1) - 2 = 7$$

Hence the simultaneous solution to equations (1) and (2) is $(7, -1)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1) \quad -2 \times 7 &= 9 \times (-1) - 5 \\ -14 &= -9 - 5 \\ -14 &= -14 \end{aligned} \qquad \begin{aligned} (2) \quad 0 &= 10 \times 7 + 20 + 90 \times (-1) \\ 0 &= 70 + 20 - 90 \\ 0 &= 0 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} -7y + 4x &= -61 \qquad (1) \\ 2y - 4x &= 46 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. We add both sides of equations (1) and (2) , giving

$$2y - 7y - 4x + 4x = 46 - 61 \qquad (3)$$

Simplifying equation (3) gives

$$\begin{aligned} -5y &= -15 \qquad (4) \\ y &= 3 \qquad (5) \end{aligned}$$

Next we substitute the value for y into equation (1) to obtain the value for x, giving

$$\begin{aligned} -7 \times 3 + 4x &= -61 \\ 4x &= -40 \qquad \text{so} \\ x &= -10 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(-10, 3)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1) \quad -7 \times 3 + 4 \times (-10) &= -61 \\ -21 - 40 &= -61 \\ -61 &= -61 \end{aligned} \qquad \begin{aligned} (2) \quad 2 \times 3 - 4 \times (-10) &= 46 \\ 6 + 40 &= 46 \\ 46 &= 46 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**4**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$0 = -27x - 3y - 3 \qquad (1)$$
$$0 = 7y + 504 - 8x \qquad (2)$$

We solve these using substitution. Rearranging equation (1) with y on the left-hand side gives

$$3y = -27x - 3 \qquad (3)$$

Dividing both sides of (3) by 3, gives

$$y = -9x - 1 \qquad (4)$$

Substituting for y in equation (2),

$$0 = 7 \times (-9x - 1) + 504 - 8x \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$\begin{aligned} 0 &= -63x - 7 + 504 - 8x \\ -497 &= -71x \\ 7 &= x \end{aligned}$$

Next we substitute the value for x into equation (4) to obtain the value for y, giving

$$y = -9 \times 7 - 1 = -64$$

Hence the simultaneous solution to equations (1) and (2) is $(7, -64)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $\begin{aligned} 0 &= -27 \times 7 - 3 \times (-64) - 3 \\ 0 &= -189 + 192 - 3 \\ 0 &= 0 \end{aligned}$

(2) $\begin{aligned} 0 &= 7 \times (-64) + 504 - 8 \times 7 \\ 0 &= -448 + 504 - 56 \\ 0 &= 0 \end{aligned}$

Both equations turned into true statements, as required. Hence the answer is correct.)

3. (**1**) First we number the equations for convenience:

$$-4y + 4x = 28 \qquad (1)$$
$$14y + 9x = -6 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 7 and equation (2) by 2 , giving

$$-28y + 28x = 196 \qquad (3)$$
$$28y + 18x = -12 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-28y + 28y + 28x + 18x = 196 - 12 \qquad (5)$$

Simplifying equation (5) gives

$$46x = 184 \qquad (6)$$
$$x = 4 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$\begin{aligned} -4y + 4 \times 4 &= 28 \\ -4y &= 12 \qquad \text{so} \\ y &= -3 \end{aligned}$$

Hence the simultaneous solution to equations (1) and (2) is $(4, -3)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\quad -4 \times (-3) + 4 \times 4 &= 28 \\ 12 + 16 &= 28 \\ 28 &= 28 \end{aligned} \qquad \begin{aligned} (2)\quad 14 \times (-3) + 9 \times 4 &= -6 \\ -42 + 36 &= -6 \\ -6 &= -6 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) First we number the equations for convenience:

$$\begin{aligned} -6x - 10y &= 116 \qquad (1) \\ 56 + 7y &= 21x \qquad (2) \end{aligned}$$

We solve these using substitution. Rearranging equation (2) with y on the left-hand side gives

$$7y = 21x - 56 \qquad (3)$$

Dividing both sides of (3) by 7, gives

$$y = 3x - 8 \qquad (4)$$

Substituting for y in equation (1),

$$-6x - 10 \times (3x - 8) = 116 \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$\begin{aligned} -6x - 30x + 80 &= 116 \\ -36x &= 36 \\ x &= -1 \end{aligned}$$

Next we substitute the value for x into equation (4) to obtain the value for y, giving

$$y = 3 \times (-1) - 8 = -11$$

Hence the simultaneous solution to equations (1) and (2) is $(-1, -11)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\quad -6 \times (-1) - 10 \times (-11) &= 116 \\ 6 + 110 &= 116 \\ 116 &= 116 \end{aligned} \qquad \begin{aligned} (2)\quad 56 + 7 \times (-11) &= 21 \times (-1) \\ 56 - 77 &= -21 \\ -21 &= -21 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$\begin{aligned} 4y + 9x &= 63 \qquad (1) \\ 6y + 7x &= 75 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by -3 and equation (2) by 2 , giving

$$-12y - 27x = -189 \qquad (3)$$
$$12y + 14x = 150 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-12y + 12y - 27x + 14x = -189 + 150 \qquad (5)$$

Simplifying equation (5) gives

$$-13x = -39 \qquad (6)$$
$$x = 3 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$4y + 9 \times 3 = 63$$
$$4y = 36 \qquad \text{so}$$
$$y = 9$$

Hence the simultaneous solution to equations (1) and (2) is $(3, 9)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $4 \times 9 + 9 \times 3 = 63$
$36 + 27 = 63$
$63 = 63$

(2) $6 \times 9 + 7 \times 3 = 75$
$54 + 21 = 75$
$75 = 75$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**4**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$0 = -10y - 80 + 60x \qquad (1)$$
$$48 + 6y = 36x \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with y on the left-hand side gives

$$6y = 36x - 48 \qquad (3)$$

Dividing both sides of (3) by 6, gives

$$y = 6x - 8 \qquad (4)$$

Substituting for y in equation (1),

$$0 = -10 \times (6x - 8) - 80 + 60x \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$0 = -60x + 80 - 80 + 60x$$
$$0 = 0$$

This statement is **always true**, so there is an infinite number of solutions to our simultaneous equations. The lines are superimposed.

4. (**1**) First we number the equations for convenience:

$$-6y - 9x = 36 \qquad (1)$$
$$-2y - 3x = 12 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (2) by -3 , giving

$$-6y - 9x = 36 \qquad (3)$$
$$6y + 9x = -36 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$6y - 6y + 9x - 9x = -36 + 36 \qquad (5)$$

Simplifying equation (5) gives

$$0 = 0 \qquad (6)$$

Statement (6) is **always true**, so there is an infinite number of solutions to our simultaneous equations.

(**2**) First we number the equations for convenience:

$$-72y = 16 + 8x \qquad (1)$$
$$-90y - 10x = 20 \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with x on the left-hand side gives

$$-10x = 90y + 20 \qquad (3)$$

Dividing both sides of (3) by -10, gives

$$x = -9y - 2 \qquad (4)$$

Substituting for x in equation (1),

$$-72y = 16 + 8 \times (-9y - 2) \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$-72y = 16 - 72y - 16$$
$$0 = 0$$

This statement is **always true**, so there is an infinite number of solutions to our simultaneous equations.

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$3y - 2x = -4 \qquad (1)$$
$$5y + 10x = 60 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 5 , giving

$$15y - 10x = -20 \qquad (3)$$
$$5y + 10x = 60 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$5y + 15y + 10x - 10x = 60 - 20 \qquad (5)$$

Simplifying equation (5) gives

$$20y = 40 \qquad (6)$$
$$y = 2 \qquad (7)$$

Next we substitute the value for y into equation (1) to obtain the value for x, giving

$$3 \times 2 - 2x = -4$$
$$-2x = -10 \qquad \text{so}$$
$$x = 5$$

Hence the simultaneous solution to equations (1) and (2) is $(5, 2)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $3 \times 2 - 2 \times 5 = -4$
$6 - 10 = -4$
$-4 = -4$

(2) $5 \times 2 + 10 \times 5 = 60$
$10 + 50 = 60$
$60 = 60$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**4**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$-10y - 3x = -36 \qquad (1)$$
$$81y = -9x - 45 \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with x on the right-hand side gives

$$81y + 45 = -9x \qquad (3)$$

Dividing both sides of (3) by -9 gives

$$-9y - 5 = x \qquad (4)$$

Substituting for x in equation (1),

$$-10y - 3 \times (-9y - 5) = -36 \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$-10y + 27y + 15 = -36$$
$$17y = -51$$
$$y = -3$$

Next we substitute the value for y into equation (4) to obtain the value for x, giving

$$x = -9 \times (-3) - 5 = 22$$

Hence the simultaneous solution to equations (1) and (2) is $(22, -3)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $-10 \times (-3) - 3 \times 22 = -36$
$30 - 66 = -36$
$-36 = -36$

(2) $81 \times (-3) = -9 \times 22 - 45$
$-243 = -198 - 45$
$-243 = -243$

Both equations turned into true statements, as required. Hence the answer is correct.)

5. (**1**) First we number the equations for convenience:

$$-2y - 9x = 55 \qquad (1)$$
$$8y - 4x = 60 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by 4 , giving

$$-8y - 36x = 220 \qquad (3)$$
$$8y - 4x = 60 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-8y + 8y - 36x - 4x = 220 + 60 \qquad (5)$$

Simplifying equation (5) gives

$$-40x = 280 \qquad (6)$$
$$x = -7 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$-2y - 9 \times (-7) = 55$$
$$-2y = -8 \qquad \text{so}$$
$$y = 4$$

Hence the simultaneous solution to equations (1) and (2) is $(-7, 4)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1) \quad -2 \times 4 - 9 \times (-7) = 55 \qquad (2) \quad 8 \times 4 - 4 \times (-7) = 60$$
$$-8 + 63 = 55 \qquad 32 + 28 = 60$$
$$55 = 55 \qquad 60 = 60$$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**2**) First we number the equations for convenience:

$$9x = 30 - 3y \qquad (1)$$
$$50 = -10y - 5x \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with x on the right-hand side gives

$$10y + 50 = -5x \qquad (3)$$

Dividing both sides of (3) by -5 gives

$$-2y - 10 = x \qquad (4)$$

Substituting for x in equation (1),

$$9 \times (-2y - 10) = 30 - 3y \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$-18y - 90 = 30 - 3y$$
$$-15y = 120$$
$$y = -8$$

Next we substitute the value for y into equation (4) to obtain the value for x, giving

$$x = -2 \times (-8) - 10 = 6$$

Hence the simultaneous solution to equations (1) and (2) is $(6, -8)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $9 \times 6 = 30 - 3 \times (-8)$
$54 = 30 + 24$
$54 = 54$

(2) $50 = -10 \times (-8) - 5 \times 6$
$50 = 80 - 30$
$50 = 50$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**3**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$5x - 8y = -37 \qquad (1)$$
$$-8x - 8y = -24 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (1) by -1 , giving

$$-5x + 8y = 37 \qquad (3)$$
$$-8x - 8y = -24 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$-5x - 8x + 8y - 8y = 37 - 24 \qquad (5)$$

Simplifying equation (5) gives

$$-13x = 13 \qquad (6)$$
$$x = -1 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$5 \times (-1) - 8y = -37$$
$$-8y = -32 \qquad \text{so}$$
$$y = 4$$

Hence the simultaneous solution to equations (1) and (2) is $(-1, 4)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

(1) $5 \times (-1) - 8 \times 4 = -37$
$-5 - 32 = -37$
$-37 = -37$

(2) $-8 \times (-1) - 8 \times 4 = -24$
$8 - 32 = -24$
$-24 = -24$

Both equations turned into true statements, as required. Hence the answer is correct.)

(**4**) We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$-32 = -4y - 24x \qquad (1)$$
$$-40 = -30x - 5y \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with y on the right-hand side gives

$$30x - 40 = -5y \qquad (3)$$

Dividing both sides of (3) by -5 gives

$$-6x + 8 = y \qquad (4)$$

Substituting for y in equation (1),

$$-32 = -4 \times (-6x + 8) - 24x \qquad (5)$$

Now (5) is an equation only involving x which gives:

$$-32 = 24x - 32 - 24x$$
$$-32 = -32$$

This statement is **always true**, so there is an infinite number of solutions to our simultaneous equations. The lines are superimposed.