Chain rule solutions.

1. $y = (2x^3+3)^6$ let $u = 2x^3+3$
$\text{so } y = u^6$ $\therefore u' = 6x^2$

$$\begin{aligned}\therefore y' &= 6u^5 \times u' \\ &= 6(2x^3+3)^5 \times 6x^2 \\ &= 36x^2(2x^3+3)^5\end{aligned}$$

2. $y = (3x+4)^4$ let $u = 3x+4$
$\text{so } y = u^4$ $\therefore u' = 3$

$$\begin{aligned}\therefore y' &= 4u^3 \times u' \\ &= 4(3x+4)^3 \times 3 \\ &= 12(3x+4)^3\end{aligned}$$

3. $y = 2(x^2+1)^5$ let $u = x^2+1$
$\text{so } y = 2u^5$ $\therefore u' = 2x$

$$\begin{aligned}\therefore y' &= 2 \times 5u^4 \times u' \\ &= 10(x^2+1)^4 \times 2x \\ &= 20x(x^2+1)^4\end{aligned}$$

4. $y = (x^3+x^2+5)^2$ let $u = x^3+x^2+5$
$\text{so } y = u^2$ $\therefore u' = 3x^2+2x$

$$\begin{aligned}\therefore y' &= 2u \times u' \\ &= 2(x^3+x^2+5) \times (3x^2+2x) \\ &= 2(x^3+x^2+5)(3x^2+2x)\end{aligned}$$

5. $y = e^{5x}$
$\therefore y' = 5e^{5x}$

or let $u = 5x$ $\therefore u' = 5$

$$\begin{aligned}\therefore y &= e^u \\ \therefore y' &= e^u \times u' \\ &= 5e^{5x}\end{aligned}$$

6. $y = e^{3x+2}$

$\therefore y' = 3e^{3x+2}$

or let $u = 3x+2 \therefore u' = 3$

so $y = e^u$

$\therefore y' = e^u \times u'$

$= 3e^{3x+2}$

7. $y = \ln(2x+4)$

so $y = \ln u$

let $u = 2x+4$

$\therefore u' = 2$

$\therefore y' = \frac{1}{u} \times u'$

$= \frac{1}{2x+4} \times 2$

$= \frac{2}{2x+4}$

$= \frac{2}{2(x+2)}$

$= \frac{1}{x+2}$

10. $y = e^{-4x-1}$

$\therefore y' = -4e^{-4x-1}$

(same as Q6).

8. $y = e^{-x^2}$

so $y = e^u$

let $u = -x^2$

$\therefore u' = -2x$

$\therefore y' = e^u \times u'$

$= e^{-x^2} \times -2x$

$= -2xe^{-x^2}$

9. $y = (3x^2+5)^{-3}$

so $y = u^{-3}$

let $u = 3x^2+5$

$\therefore u' = 6x$

$\therefore y' = -3u^{-4} \times u'$

$= -3(3x^2+5)^{-4} \times 6x$

$= -18x(3x^2+5)^{-4}$