

Find and classify all critical points of the following equations. You should find both x and y-coordinates.

1. $y = 3x^2 + 12x - 8$

$$y' = 6x + 12$$

$$6x + 12 = 0$$

$$6x = -12$$

$$x = -2$$

sub $x = -2$ into y

$$\Rightarrow y = 3x^2 + 12x - 8$$

$$= 3 \times 4 - 24 - 8$$

$$= 12 - 24 - 8$$

$$= -20 \Rightarrow (-2, -20)$$

$$y'' = 6 \text{ which is } \underline{\underline{+ve}}$$

$\therefore (-2, -20)$ is a minimum

2. $y = -x^3 + 9x^2 - 24x - 3$

$$y' = -3x^2 + 18x - 24$$

$$-3x^2 + 18x - 24 = 0 \quad (\div 3)$$

$$-x^2 + 6x - 8 = 0$$

$$a = -1 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = 6$$

$$c = -8$$

$$= \frac{-6 \pm \sqrt{6^2 - 4(-1)(-8)}}{2(-1)}$$

$$= \frac{-6 \pm \sqrt{36 - 32}}{-2}$$

$$= \frac{-6 \pm \sqrt{4}}{-2}$$

$$= \frac{-6 \pm 2}{-2}$$

$$= \frac{-6+2}{-2} \text{ or } \frac{-6-2}{-2}$$

$$= \frac{-4}{-2} \text{ or } \frac{-8}{-2}$$

$$= 2 \text{ or } 4$$

sub $x = 2$ into y

$$\Rightarrow y = -3x^2 + 18x - 24$$

$$= -3 \times 4 + 36 - 24$$

$$= -12 + 36 - 24$$

$$= 0$$

$$\Rightarrow (2, 0)$$

sub $x = 4$ into y

$$\Rightarrow y = -3x^2 + 18x - 24$$

$$= -48 + 72 - 24$$

$$= 0$$

$$\Rightarrow (4, 0)$$

$$y'' = -6x + 18$$

$$\text{sub } x = 2 \text{ in } y'' \Rightarrow -6 \times 2 + 18 = 6 \therefore \text{MIN}$$

$$\text{sub } x = 4 \text{ in } y'' \Rightarrow -6 \times 4 + 18 = -6 \therefore \text{MAX}$$

A question on functions and derivatives (previous exam question)

Let $y = -x^2 + x + 1$

(a) Find y'

(b) Find the x-coordinate(s) of the point(s) at which the value of the function equals the slope of the function

a) $y' = -2x + 1$

b) $y = y'$

$\Rightarrow -x^2 + x + 1 = -2x + 1$

$-x^2 + x + 1 + 2x - 1 = 0$

$-x^2 + 3x = 0$

$-x(x-3) = 0$

$\underline{x=0}$ or $x-3=0$

$\underline{x=3}$

OR

$a = -1$

$b = 3$

$c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2 - 4(-1)(0)}}{2(-1)}$

$= \frac{-3 \pm \sqrt{9}}{-2}$

$= \frac{-3 \pm 3}{-2}$

$= \frac{-3+3}{-2}$ OR $\frac{-3-3}{2}$

$= \underline{\underline{0}}$

$= \underline{\underline{-3}}$