MATH1040 Solutions to Practice Questions 3

1. **(a)** $\dfrac{x^0x^1}{x^2x^2} = \dfrac{x^{0+1}}{x^{2+2}} = \dfrac{x^1}{x^4} = x^{1-4} = x^{-3}$.

(b)
$$\begin{aligned} x^0x^{-3}y^{-3} \div (x^{-2}y^{-1}) \times y^1 &= x^0x^{-3}y^{-3} \times x^2y^1 \times y^1 \\ &= x^{0-3+2}y^{-3+1+1} \\ &= x^{-1}y^{-1}. \end{aligned}$$

(c)
$$\begin{aligned} -3x + 5 &\leq -2x - 4 \\ -3x + 5 - 5 &\leq -2x - 4 - 5 \\ -3x &\leq -2x - 9 \\ -3x + 2x &\leq -2x - 9 + 2x \\ -x &\leq -9 \\ -x \div -1 &\geq -9 \div -1 \qquad \text{(reversing the inequality sign)} \\ x &\geq 9. \end{aligned}$$

In inequality form, the answer is $[9, \infty)$.

(d) $\displaystyle\sum_{i=2}^{6} 3ix = 6x + 9x + 12x + 15x + 18x = 60x.$

(e)
$$\begin{aligned} \sum_{i=1}^{5}(-1)^i i &= (-1)^1 \times 1 + (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 \\ &= -1 + 2 - 3 + 4 - 5 = -3. \end{aligned}$$

(f) $\dfrac{-4}{2} + \dfrac{-4}{3} + \dfrac{-4}{4} + \dfrac{-4}{5} = \displaystyle\sum_{i=2}^{5} \frac{-4}{i}$.

(g) $\displaystyle\sum_{i=2}^{x} -2i = -28$, so trial and error gives $x = 5$.

(h) $\displaystyle\sum_{i=x}^{-2} -2i = 18$, so trial and error gives $x = -4$.

(i) $\displaystyle\sum_{i=x-1}^{x} -2i = 14$, so $-2(x-1) - 2x = 14$, so $2 - 4x = 14$, so $-4x = 12$, so $x = -3$.

(j) $\displaystyle\sum_{i=-1}^{2} xi = -2$, so $-x + 0 + x + 2x = -2$, so $2x = -2$, so $x = -1$.

(k) $\displaystyle\sum_{i=0}^{1} -2x = 8$, so $-2x - 2x = 8$, so $-4x = 8$, so $x = -2$.

(l) $x = \displaystyle\sum_{i=1}^{3} 2i$, so $x = (2 \times 1) + (2 \times 2) + (2 \times 3)$, so $x = 2 + 4 + 6$, so $x = 12$.

2. **(a)** $\dfrac{x^0x^{-1}}{x^{-2}x^3} = \dfrac{x^{0-1}}{x^{-2+3}} = \dfrac{x^{-1}}{x^1} = x^{-1-1} = x^{-2}$.

(b)
$$\begin{aligned} x^{-3}x^0y^{-2} \div (x^{-2}y^{-3}) \times y^2 &= x^{-3}x^0y^{-2} \times x^2y^3 \times y^2 \\ &= x^{-3+0+2}y^{-2+3+2} \\ &= x^{-1}y^3. \end{aligned}$$

(c)
$$\begin{aligned} -2x+4 &\geq 3x-5 \\ -2x+4-4 &\geq 3x-5-4 \\ -2x &\geq 3x-9 \\ -2x-3x &\geq 3x-9-3x \\ -5x &\geq -9 \\ -5x \div -5 &\leq -9 \div -5 \qquad \text{(reversing the inequality sign)} \\ x &\leq \frac{9}{5}. \end{aligned}$$

In inequality form, the answer is $(-\infty, \frac{9}{5}]$.

(d) $\sum_{i=-2}^{1} 2ix = -4x - 2x + 0x + 2x = -4x.$

(e)
$$\begin{aligned} \sum_{i=1}^{4}(-1)^i i &= (-1)^1 \times 1 + (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 \\ &= -1 + 2 - 3 + 4 = 2. \end{aligned}$$

(f) $\frac{-1}{4} + \frac{-1}{5} + \frac{-1}{6} + \frac{-1}{7} + \frac{-1}{8} + \frac{-1}{9} = \sum_{i=4}^{9} \frac{-1}{i}.$

(g) $\sum_{i=2}^{x} i = 9$, so trial and error gives $x = 4$.

(h) $\sum_{i=x}^{-2} -i = 5$, so trial and error gives $x = -3$.

(i) $\sum_{i=x-1}^{x} i = 5$, so $(x-1) + x = 5$, so $-1 + 2x = 5$, so $2x = 6$, so $x = 3$.

(j) $\sum_{i=-1}^{0} xi = -2$, so $-x + 0 = -2$, so $-x = -2$, so $x = 2$.

(k) $\sum_{i=-2}^{0} x = 3$, so $x + x + x = 3$, so $3x = 3$, so $x = 1$.

(l) $x = \sum_{i=1}^{4} 3i$, so $x = (3 \times 1) + (3 \times 2) + (3 \times 3) + (3 \times 4)$, so $x = 3 + 6 + 9 + 12$, so $x = 30$.

3. (a) $\frac{x^3 x^{-1}}{x^1 x^0} = \frac{x^{3-1}}{x^{1+0}} = \frac{x^2}{x^1} = x^{2-1} = x^1.$

(b)
$$\begin{aligned} x^{-3}x^2y^3 \div (x^{-1}y^{-2}) \times y^2 &= x^{-3}x^2y^3 \times x^1y^2 \times y^2 \\ &= x^{-3+2+1}y^{3+2+2} \\ &= x^0y^7 \\ &= y^7. \end{aligned}$$

(c)
$$\begin{aligned} -2x+3 &\geq 2x-4 \\ -2x+3-3 &\geq 2x-4-3 \\ -2x &\geq 2x-7 \\ -2x-2x &\geq 2x-7-2x \\ -4x &\geq -7 \\ -4x \div -4 &\leq -7 \div -4 \qquad \text{(reversing the inequality sign)} \\ x &\leq \frac{7}{4}. \end{aligned}$$

In inequality form, the answer is $(-\infty, \frac{7}{4}]$.

(d) $\sum_{i=2}^{5} 3ix = 6x + 9x + 12x + 15x = 42x.$

(e)
$$\begin{aligned}\sum_{i=2}^{5}(-1)^i i &= (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 \\ &= 2 - 3 + 4 - 5 = -2.\end{aligned}$$

(f) $\frac{4}{4} + \frac{4}{5} + \frac{4}{6} + \frac{4}{7} + \frac{4}{8} + \frac{4}{9} = \sum_{i=4}^{9} \frac{4}{i}.$

(g) $\sum_{i=2}^{x} i = 2$, so trial and error gives $x = 2$.

(h) $\sum_{i=x}^{0} -2i = 2$, so trial and error gives $x = -1$.

(i) $\sum_{i=x-1}^{x} -i = 1$, so $-(x-1) - x = 1$, so $1 - 2x = 1$, so $-2x = 0$, so $x = 0$.

(j) $\sum_{i=-2}^{-1} xi = 3$, so $-2x - x = 3$, so $-3x = 3$, so $x = -1$.

(k) $\sum_{i=1}^{2} x = -2$, so $x + x = -2$, so $2x = -2$, so $x = -1$.

(l) $x = \sum_{i=1}^{3} 3i$, so $x = (3 \times 1) + (3 \times 2) + (3 \times 3)$, so $x = 3 + 6 + 9$, so $x = 18$.

4. (a) $\frac{x^{-1}x^3}{x^2x^1} = \frac{x^{-1+3}}{x^{2+1}} = \frac{x^2}{x^3} = x^{2-3} = x^{-1}.$

(b)
$$\begin{aligned}x^{-1}x^{-1}y^{-3} \div (x^{-3}y^{-1}) \times y^0 &= x^{-1}x^{-1}y^{-3} \times x^3y^1 \times y^0 \\ &= x^{-1-1+3}y^{-3+1+0} \\ &= x^1y^{-2}.\end{aligned}$$

(c)
$$\begin{aligned}-2x + 5 &\leq -3x - 4 \\ -2x + 5 - 5 &\leq -3x - 4 - 5 \\ -2x &\leq -3x - 9 \\ -2x + 3x &\leq -3x - 9 + 3x \\ x &\leq -9.\end{aligned}$$

In inequality form, the answer is $(-\infty, -9]$.

(d) $\sum_{i=0}^{3} 4ix = 0x + 4x + 8x + 12x = 24x.$

(e)
$$\begin{aligned}\sum_{i=2}^{5}(-1)^i i &= (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 \\ &= 2 - 3 + 4 - 5 = -2.\end{aligned}$$

(f) $\frac{-3}{3} + \frac{-3}{4} + \frac{-3}{5} + \frac{-3}{6} + \frac{-3}{7} + \frac{-3}{8} + \frac{-3}{9} = \sum_{i=3}^{9} \frac{-3}{i}.$

(g) $\sum_{i=2}^{x} -i = -5$, so trial and error gives $x = 3$.

(h) $\sum_{i=x}^{-1} 3i = -9$, so trial and error gives $x = -2$.

(i) $\sum_{i=x-1}^{x} 3i = -15$, so $3(x-1) + 3x = -15$, so $-3 + 6x = -15$, so $6x = -12$, so $x = -2$.

(j) $\sum_{i=1}^{4} xi = 0$, so $x + 2x + 3x + 4x = 0$, so $10x = 0$, so $x = 0$.

(k) $\sum_{i=1}^{4} -x = 4$, so $-x - x - x - x = 4$, so $-4x = 4$, so $x = -1$.

(l) $x = \sum_{i=1}^{3} i^0$, so $x = 1^0 + 2^0 + 3^0$, so $x = 1 + 1 + 1$, so $x = 3$.

5. **(a)** $\dfrac{x^1 x^{-3}}{x^{-2} x^1} = \dfrac{x^{1-3}}{x^{-2+1}} = \dfrac{x^{-2}}{x^{-1}} = x^{-2-(-1)} = x^{-1}$.

(b)
$$\begin{aligned} x^1 x^{-1} y^3 \div (x^2 y^2) \times y^{-3} &= x^1 x^{-1} y^3 \times x^{-2} y^{-2} \times y^{-3} \\ &= x^{1-1-2} y^{3-2-3} \\ &= x^{-2} y^{-2}. \end{aligned}$$

(c)
$$\begin{aligned} 3x + 3 &> -2x - 1 \\ 3x + 3 - 3 &> -2x - 1 - 3 \\ 3x &> -2x - 4 \\ 3x + 2x &> -2x - 4 + 2x \\ 5x &> -4 \\ 5x \div 5 &> -4 \div 5 \\ x &> -\frac{4}{5}. \end{aligned}$$

In inequality form, the answer is $(-\frac{4}{5}, \infty)$.

(d) $\sum_{i=0}^{3} 2ix = 0x + 2x + 4x + 6x = 12x$.

(e)
$$\begin{aligned} \sum_{i=1}^{5} (-1)^i i &= (-1)^1 \times 1 + (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 \\ &= -1 + 2 - 3 + 4 - 5 = -3. \end{aligned}$$

(f) $\dfrac{1}{-8} + \dfrac{1}{-7} + \dfrac{1}{-6} + \dfrac{1}{-5} + \dfrac{1}{-4} + \dfrac{1}{-3} + \dfrac{1}{-2} = \sum_{i=-8}^{-2} \dfrac{1}{i}$.

(g) $\sum_{i=1}^{x} -2i = -2$, so trial and error gives $x = 1$.

(h) $\sum_{i=x}^{-1} 3i = -18$, so trial and error gives $x = -3$.

(i) $\sum_{i=x-1}^{x} 2i = 10$, so $2(x-1) + 2x = 10$, so $-2 + 4x = 10$, so $4x = 12$, so $x = 3$.

(j) $\sum_{i=2}^{4} xi = 9, \quad \text{so } 2x + 3x + 4x = 9, \quad \text{so } 9x = 9, \quad \text{so } x = 1.$

(k) $\sum_{i=1}^{2} x = 2, \quad \text{so } x + x = 2, \quad \text{so } 2x = 2, \quad \text{so } x = 1.$

(l) $x = \sum_{i=-1}^{0} 3i^2, \quad \text{so } x = (3 \times -1^2) + (3 \times 0^2), \quad \text{so } x = 3 + 0, \quad \text{so } x = 3.$