MATH1040 Solutions to Practice Questions 4

1. (a) (1) $A = \{6, 7, 8, 9, 10, 11\}$.

(2) $A \cup B = \{5, 6, 7, 8, 9, 10, 11, 13, 15\}$.

(3) $B \cap C = \{6\}$.

(4) $A \setminus C = \{7, 9, 10, 11\}$.

(5) $A \setminus (B \cup C) = \{7, 9, 10, 11\}$.

(6) $(A \cap B) \cap C = \{6\}$.

(b) (1) $B \cup C = \{-5, -3, -1, 1\} \cup \{-2, 3, 5\} = \{-5, -3, -2, -1, 1, 3, 5\}$

(2) $C \setminus \emptyset = \{-2, 3, 5\} \setminus \{\} = \{-2, 3, 5\}$

(3)
$$\begin{aligned} C \setminus (B \cap A) &= C \setminus (\{-5, -3, -1, 1\} \cap \{-1, 2, 3\}) \\ &= \{-2, 3, 5\} \setminus \{-1\} = \{-2, 3, 5\} \end{aligned}$$

(4)
$$\begin{aligned} (C \setminus A) \setminus B &= (\{-2, 3, 5\} \setminus \{-1, 2, 3\}) \setminus B \\ &= \{-2, 5\} \setminus \{-5, -3, -1, 1\} = \{-2, 5\} \end{aligned}$$

(5)
$$\begin{aligned} (B \cap \emptyset) \cup C &= (\{-5, -3, -1, 1\} \cap \{\}) \cup C \\ &= \{\} \cup \{-2, 3, 5\} = \{-2, 3, 5\} \end{aligned}$$

(6)
$$\begin{aligned} (A \cup B) \cup (B \cup C) &= (\{-1, 2, 3\} \cup \{-5, -3, -1, 1\}) \cup (\{-5, -3, -1, 1\} \cup \{-2, 3, 5\}) \\ &= \{-5, -3, -1, 1, 2, 3\} \cup \{-5, -3, -2, -1, 1, 3, 5\} = \{-5, -3, -2, -1, 1, 2, 3, 5\}. \end{aligned}$$

(7)
$$\begin{aligned} (\emptyset \cap C) \cap (B \cap A) &= (\{\} \cap \{-2, 3, 5\}) \cap (\{-5, -3, -1, 1\} \cap \{-1, 2, 3\}) \\ &= \{\} \cap \{-1\} = \{\}. \end{aligned}$$

(c) (1) $p = 5/10 = 1/2$.
(2) $p = 4/10 = 2/5$.
(3) $p = 2/10 = 1/5$.
(4) $p = 7/10$.
(5) $p = 2/4 = 1/2$.
(6) Prob(r_1 is even) equals 5/10, and Prob(r_2 is even) equals 3/7. Now r_1 and r_2 are chosen independently, so Prob(both even) = Prob(r_1 is even) $\times$ Prob(r_2 is even).

Hence $p = \frac{5}{10} \times \frac{3}{7} = \frac{15}{70} = \frac{3}{14}$.
(7) By the principle of inclusion/exclusion,

Prob(r_1 even **or** r_2 even) = Prob(r_1 even)+Prob(r_2 even)− Prob(r_1 **and** r_2 are even).

Hence $p = \frac{5}{10} + \frac{3}{7} - \frac{15}{70} = \frac{35 + 30 - 15}{70} = \frac{50}{70} = \frac{5}{7}$.
(8) Now r_1 and r_2 are chosen independently, so

Prob(r_1 is even **given that** r_2 is even) =Prob(r_1 is even).

Hence $p = 5/10 = 1/2$.

2. (a) (1) $A = \{2, 3, 4, 5, 6, 7\}$.

(2) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 10, 12\}$.

(3) $B \cap C = \{2\}$.

(4) $A \setminus C = \{3, 5, 7\}$.

(5) $A \setminus (B \cup C) = \{3, 5, 7\}$.

(6) $(A \cap B) \cap C = \{2\}$.

(b) (1) $C \cap B = \{-2,-1,1\} \cap \{-5,-3,-2\} = \{-2\}$

(2) $A \cup \emptyset = \{-3,1,3,5\} \cup \{\} = \{-3,1,3,5\}$

(3)
$$\begin{aligned} A \cup (C \cup B) &= A \cup (\{-2,-1,1\} \cup \{-5,-3,-2\}) \\ &= \{-3,1,3,5\} \cup \{-5,-3,-2,-1,1\} = \{-5,-3,-2,-1,1,3,5\} \end{aligned}$$

(4)
$$\begin{aligned} (C \cup B) \setminus A &= (\{-2,-1,1\} \cup \{-5,-3,-2\}) \setminus A \\ &= \{-5,-3,-2,-1,1\} \setminus \{-3,1,3,5\} = \{-5,-2,-1\} \end{aligned}$$

(5)
$$\begin{aligned} (A \setminus B) \cap \emptyset &= (\{-3,1,3,5\} \setminus \{-5,-3,-2\}) \cap \emptyset \\ &= \{1,3,5\} \cap \{\} = \{\} \end{aligned}$$

(6)
$$\begin{aligned} (C \cup B) \setminus (B \cup A) &= (\{-2,-1,1\} \cup \{-5,-3,-2\}) \setminus (\{-5,-3,-2\} \cup \{-3,1,3,5\}) \\ &= \{-5,-3,-2,-1,1\} \setminus \{-5,-3,-2,1,3,5\} = \{-1\}. \end{aligned}$$

(7)
$$\begin{aligned} (B \cap \emptyset) \cup (C \setminus \emptyset) &= (\{-5,-3,-2\} \cap \{\}) \cup (\{-2,-1,1\} \setminus \{\}) \\ &= \{\} \cup \{-2,-1,1\} = \{-2,-1,1\}. \end{aligned}$$

(c) (1) $p = 4/9$.

(2) $p = 7/9$.

(3) $p = 3/9 = 1/3$.

(4) $p = 8/9$.

(5) $p = 3/7$.

(6) Prob(r_1 is even) equals 4/9, and Prob(r_2 is even) equals 3/6. Now r_1 and r_2 are chosen independently, so Prob(both even) = Prob(r_1 is even) $\times$ Prob(r_2 is even).

Hence $p = \frac{4}{9} \times \frac{3}{6} = \frac{12}{54} = \frac{2}{9}$.

(7) By the principle of inclusion/exclusion,

Prob(r_1 even **or** r_2 even) = Prob(r_1 even)+Prob(r_2 even)− Prob(r_1 **and** r_2 are even).

Hence $p = \frac{4}{9} + \frac{3}{6} - \frac{12}{54} = \frac{24+27-12}{54} = \frac{39}{54} = \frac{13}{18}$.

(8) Now r_1 and r_2 are chosen independently, so

Prob(r_1 is even **given that** r_2 is even) =Prob(r_1 is even).

Hence $p = 4/9$.

3. (a) (1) $A = \{6,7,8,9,10\}$.

(2) $A \cup B = \{5,6,7,8,9,10,12,14\}$.

(3) $B \cap C = \{6\}$.

(4) $A \setminus C = \{7,9,10\}$.

(5) $A \setminus (B \cup C) = \{7,9,10\}$.

(6) $(A \cap B) \cap C = \{6\}$.

(b) (1) $A \setminus B = \{-1,1,2,4\} \setminus \{-5,-3,2,4\} = \{-1,1\}$

(2) $C \cap \emptyset = \{-5,-2,-1\} \cap \{\} = \{\}$

(3)
$$\begin{aligned} B \cap (C \cup A) &= B \cap (\{-5,-2,-1\} \cup \{-1,1,2,4\}) \\ &= \{-5,-3,2,4\} \cap \{-5,-2,-1,1,2,4\} = \{-5,2,4\} \end{aligned}$$

(4)
$$\begin{aligned} (C \setminus B) \cup A &= (\{-5,-2,-1\} \setminus \{-5,-3,2,4\}) \cup A \\ &= \{-2,-1\} \cup \{-1,1,2,4\} = \{-2,-1,1,2,4\} \end{aligned}$$

(5)
$$(\emptyset \cap C) \cup A = (\{\} \cap \{-5,-2,-1\}) \cup A = \{\} \cup \{-1,1,2,4\} = \{-1,1,2,4\}$$

(6)
$$(A \setminus C) \cap (B \cap C) = (\{-1,1,2,4\} \setminus \{-5,-2,-1\}) \cap (\{-5,-3,2,4\} \cap \{-5,-2,-1\}) = \{1,2,4\} \cap \{-5\} = \{\}.$$

(7)
$$(\emptyset \setminus B) \cup (A \cap \emptyset) = (\{\} \setminus \{-5,-3,2,4\}) \cup (\{-1,1,2,4\} \cap \{\}) = \{\} \cup \{\} = \{\}.$$

(c) (1) $p = 4/8 = 1/2$.
(2) $p = 4/8 = 1/2$.
(3) $p = 2/8 = 1/4$.
(4) $p = 6/8 = 3/4$.
(5) $p = 2/4 = 1/2$.
(6) Prob(r_1 is even) equals 4/8, and Prob(r_2 is even) equals 5/10. Now r_1 and r_2 are chosen independently, so Prob(both even) = Prob(r_1 is even) × Prob(r_2 is even).

Hence $p = \frac{4}{8} \times \frac{5}{10} = \frac{20}{80} = \frac{1}{4}$.
(7) By the principle of inclusion/exclusion,
Prob(r_1 even **or** r_2 even) = Prob(r_1 even)+Prob(r_2 even)− Prob(r_1 **and** r_2 are even).
Hence $p = \frac{4}{8} + \frac{5}{10} - \frac{20}{80} = \frac{40+40-20}{80} = \frac{60}{80} = \frac{3}{4}$.
(8) Now r_1 and r_2 are chosen independently, so
Prob(r_1 is even **given that** r_2 is even) =Prob(r_1 is even).
Hence $p = 4/8 = 1/2$.

4. (a) (1) $A = \{5,6,7,8\}$.

(2) $A \cup B = \{4,5,6,7,8,9\}$.

(3) $B \cap C = \{4\}$.

(4) $A \setminus C = \{5,7\}$.

(5) $A \setminus (B \cup C) = \{\}$.

(6) $(A \cap B) \cap C = \{\}$.

(b) (1) $C \setminus A = \{-4,0,2,4\} \setminus \{-5,-2,5\} = \{-4,0,2,4\}$

(2) $A \cup \emptyset = \{-5,-2,5\} \cup \{\} = \{-5,-2,5\}$

(3)
$$C \cup (A \cap B) = C \cup (\{-5,-2,5\} \cap \{-5,-1,1,4\}) = \{-4,0,2,4\} \cup \{-5\} = \{-5,-4,0,2,4\}$$

(4)
$$(A \cap B) \cap C = (\{-5,-2,5\} \cap \{-5,-1,1,4\}) \cap C = \{-5\} \cap \{-4,0,2,4\} = \{\}$$

(5)
$$(\emptyset \cap A) \cup B = (\{\} \cap \{-5,-2,5\}) \cup B = \{\} \cup \{-5,-1,1,4\} = \{-5,-1,1,4\}$$

(6)
$$(A \cap C) \setminus (C \cap A) = (\{-5,-2,5\} \cap \{-4,0,2,4\}) \setminus (\{-4,0,2,4\} \cap \{-5,-2,5\}) = \{\} \setminus \{\} = \{\}.$$

(7)
$$(C \cap B) \cap (B \setminus \emptyset) = (\{-4,0,2,4\} \cap \{-5,-1,1,4\}) \cap (\{-5,-1,1,4\} \setminus \{\}) = \{4\} \cap \{-5,-1,1,4\} = \{4\}.$$

(c) (1) $p = 4/8 = 1/2$.
(2) $p = 6/8 = 3/4$.
(3) $p = 3/8$.
(4) $p = 7/8$.
(5) $p = 3/6 = 1/2$.
(6) Prob(r_1 is even) equals 4/8, and Prob(r_2 is even) equals 4/9. Now r_1 and r_2 are chosen independently, so Prob(both even) = Prob(r_1 is even) $\times$ Prob(r_2 is even).

Hence $p = \frac{4}{8} \times \frac{4}{9} = \frac{16}{72} = \frac{2}{9}$.
(7) By the principle of inclusion/exclusion,

Prob(r_1 even **or** r_2 even) = Prob(r_1 even)+Prob(r_2 even)− Prob(r_1 **and** r_2 are even).

Hence $p = \frac{4}{8} + \frac{4}{9} - \frac{16}{72} = \frac{36+32-16}{72} = \frac{52}{72} = \frac{13}{18}$.
(8) Now r_1 and r_2 are chosen independently, so

Prob(r_1 is even **given that** r_2 is even) =Prob(r_1 is even).

Hence $p = 4/8 = 1/2$.

5. (a) (1) $A = \{5, 6, 7, 8, 9, 10\}$.

(2) $A \cup B = \{4, 5, 6, 7, 8, 9, 10, 11, 13\}$.

(3) $B \cap C = \{4\}$.

(4) $A \setminus C = \{5, 7, 9, 10\}$.

(5) $A \setminus (B \cup C) = \{7, 9, 10\}$.

(6) $(A \cap B) \cap C = \{\}$.

(b) (1) $A \setminus B = \{-4, 0, 5\} \setminus \{-2, 0, 1, 3\} = \{-4, 5\}$

(2) $\emptyset \cup A = \{\} \cup \{-4, 0, 5\} = \{-4, 0, 5\}$

(3)
$$\begin{aligned} A \setminus (C \cap B) &= A \setminus (\{-5, -3, 0\} \cap \{-2, 0, 1, 3\}) \\ &= \{-4, 0, 5\} \setminus \{0\} = \{-4, 5\} \end{aligned}$$

(4)
$$\begin{aligned} (B \cup C) \setminus A &= (\{-2, 0, 1, 3\} \cup \{-5, -3, 0\}) \setminus A \\ &= \{-5, -3, -2, 0, 1, 3\} \setminus \{-4, 0, 5\} = \{-5, -3, -2, 1, 3\} \end{aligned}$$

(5)
$$\begin{aligned} (C \cap \emptyset) \cap B &= (\{-5, -3, 0\} \cap \{\}) \cap B \\ &= \{\} \cap \{-2, 0, 1, 3\} = \{\} \end{aligned}$$

(6)
$$\begin{aligned} (A \setminus C) \setminus (A \setminus C) &= (\{-4, 0, 5\} \setminus \{-5, -3, 0\}) \setminus (\{-4, 0, 5\} \setminus \{-5, -3, 0\}) \\ &= \{-4, 5\} \setminus \{-4, 5\} = \{\}. \end{aligned}$$

(7)
$$\begin{aligned} (A \setminus B) \cup (\emptyset \cap C) &= (\{-4, 0, 5\} \setminus \{-2, 0, 1, 3\}) \cup (\{\} \cap \{-5, -3, 0\}) \\ &= \{-4, 5\} \cup \{\} = \{-4, 5\}. \end{aligned}$$

(c) (1) $p = 3/6 = 1/2$.
(2) $p = 5/6$.
(3) $p = 3/6 = 1/2$.
(4) $p = 5/6$.
(5) $p = 3/5$.
(6) Prob(r_1 is even) equals 3/6, and Prob(r_2 is even) equals 5/10. Now r_1 and r_2 are chosen independently, so Prob(both even) = Prob(r_1 is even) $\times$ Prob(r_2 is even).

Hence $p = \frac{3}{6} \times \frac{5}{10} = \frac{15}{60} = \frac{1}{4}$.

(7) By the principle of inclusion/exclusion,

$$\text{Prob}(r_1 \text{ even } \textbf{or } r_2 \text{ even}) = \text{Prob}(r_1 \text{ even}) + \text{Prob}(r_2 \text{ even}) - \text{Prob}(r_1 \textbf{ and } r_2 \text{ are even}).$$

Hence $p = \dfrac{3}{6} + \dfrac{5}{10} - \dfrac{15}{60} = \dfrac{30+30-15}{60} = \dfrac{45}{60} = \dfrac{3}{4}$.

(8) Now r_1 and r_2 are chosen independently, so

$$\text{Prob}(r_1 \text{ is even } \textbf{given that } r_2 \text{ is even}) = \text{Prob}(r_1 \text{ is even}).$$

Hence $p = 3/6 = 1/2$.