MATH1040 Solutions to Practice Questions 5

1. (a) Let $(x_1, y_1) = (-3, -3)$ and $(x_2, y_2) = (2, 1)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so

$d = \sqrt{(-3-2)^2 + (-3-1)^2} = \sqrt{-5^2 + -4^2} = \sqrt{25+16} = \sqrt{41}$.

(b) $-y - 3x - 2 = -2$, so $-y = -2 + 3x + 2$, so $-y = 3x$, so $y = -3x$. Hence the gradient is $m = -3$ and the y-intercept is $c = 0$.

(c) Thus the equation of the line is $y = 2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (5, -5)$ into this equation to get the value for c.

Hence $-5 = 2 \times 5 + c$, so $-5 = 10 + c$. Hence $c = -5 - 10 = -15$.

Hence the equation of the line is $y = 2x - 15$.

(d) Let $(x_1, y_1) = (0, -4)$ and $(x_2, y_2) = (0, -1)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1+4}{0-0} = \dfrac{3}{0}$. This is a vertical line.

Hence the equation of the line is $x = 0$.

(e) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\square$ instead of any constant term(s).

Now $3y + x - 1 = -1$, so $3y = -x + \square$. Dividing each side by 3 gives $y = \dfrac{-1}{3}x + \square$, so $y = -\dfrac{1}{3}x + \square$. Hence the gradient of the original line is $m = -\dfrac{1}{3}$.

The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = -\dfrac{1}{3}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-1, -1)$ into this equation to get the value for c.

Hence $-1 = -\dfrac{1}{3} \times -1 + c$, so $-1 = \dfrac{1}{3} + c$. Hence $c = -1 - \dfrac{1}{3} = -\dfrac{4}{3}$.

Hence the equation of the line is $y = -\dfrac{1}{3}x - \dfrac{4}{3}$.

(f) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\square$ instead of any constant term(s).

Now $-3y + 2x - 3 = -3$, so $-3y = -2x + \square$. Dividing each side by -3 gives $y = \dfrac{-2}{-3}x + \square$, so $y = \dfrac{2}{3}x + \square$. Hence the gradient of the original line is $m = \dfrac{2}{3}$.

The new line is perpendicular to the original line, so the product of their gradients must equal -1. Let m be the gradient of the new line, so $m \times \dfrac{2}{3} = -1$. Hence $m = -\dfrac{3}{2}$. Thus the equation of the line is $y = -\dfrac{3}{2}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (4, -2)$ into this equation to get the value for c.

Hence $-2 = -\dfrac{3}{2} \times 4 + c$, so $-2 = \dfrac{-12}{2} + c$, so $-2 = -6 + c$. Hence $c = -2 + 6 = 4$.

Hence the equation of the line is $y = -\dfrac{3}{2}x + 4$.

2. (a) Let $(x_1, y_1) = (3, -4)$ and $(x_2, y_2) = (3, -3)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so

$d = \sqrt{(3-3)^2 + (-4+3)^2} = \sqrt{0^2 + -1^2} = \sqrt{0+1} = \sqrt{1} = 1$.

(b) $-3y - 3x + 2 = -2$, so $-3y = -2 + 3x - 2$, so $-3y = 3x - 4$, so $3y = -3x + 4$. Then dividing each side by 3 gives $y = \dfrac{-3}{3}x + \dfrac{4}{3} = -x + \dfrac{4}{3}$. Hence the gradient is $m = -1$ and the y-intercept is $c = \dfrac{4}{3}$.

(c) The line is horizontal, and as the point $(2, 0)$ is on the line, the equation of the line is $y = 0$.

(d) Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (4, 0)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-4}{4-2} = \dfrac{-4}{2} = -2$. Hence $m = -2$.

Thus the equation of the line is $y = -2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (2, 4)$ into this equation to get the value for c.

Hence $4 = -2 \times 2 + c$, so $4 = -4 + c$. Hence $c = 4 + 4 = 8$.

Hence the equation of the line is $y = -2x + 8$.

(e) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $3x + 1 = 2$, so $3x = \Box$, so $x = \Box$. The original line is vertical, so its gradient is undefined.

The new line is parallel to the original line, so it has the same gradient as the original line. The point $(3, 0)$ lies on the new line, so the equation of the new line is $x = 3$.

(f) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $-3y - 3x - 2 = -1$, so $-3y = 3x + \Box$. Dividing each side by -3 gives $y = \dfrac{3}{-3}x + \Box$, so $y = -x + \Box$. Hence the gradient of the original line is $m = -1$.

The new line is perpendicular to the original line, so the product of their gradients must equal -1. Let m be the gradient of the new line, so $m \times -1 = -1$. Hence $m = 1$. Thus the equation of the line is $y = x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (5, -2)$ into this equation to get the value for c.

Hence $-2 = 1 \times 5 + c$, so $-2 = 5 + c$. Hence $c = -2 - 5 = -7$.

Hence the equation of the line is $y = x - 7$.

3. (a) Let $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 4)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so

$d = \sqrt{(-3-3)^2 + (-2-4)^2} = \sqrt{-6^2 + -6^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$.

(b) $3x - 2 = -3$, so $3x = -1$. Then dividing each side by 3 gives $x = \dfrac{-1}{3} = -\dfrac{1}{3}$. Hence the line is vertical, so the gradient is undefined and there is no y-intercept.

(c) Thus the equation of the line is $y = -4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (0, 3)$ into this equation to get the value for c.

Hence $3 = -4 \times 0 + c$, so $3 = c$.

Hence the equation of the line is $y = -4x + 3$.

(d) Let $(x_1, y_1) = (3, 3)$ and $(x_2, y_2) = (-3, -1)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-3}{-3-3} = \dfrac{-4}{-6} = \dfrac{2}{3}$. Hence $m = \dfrac{2}{3}$.

Thus the equation of the line is $y = \dfrac{2}{3}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (3, 3)$ into this equation to get the value for c.

Hence $3 = \dfrac{2}{3} \times 3 + c$, so $3 = \dfrac{6}{3} + c$, so $3 = 2 + c$. Hence $c = 3 - 2 = 1$.

Hence the equation of the line is $y = \frac{2}{3}x + 1$.

(e) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $3y + 3 = 1$, so $3y = \Box$, so $y = \Box$. The original line is horizontal, so has gradient equal to 0.

The new line is parallel to the original line, so it has the same gradient as the original line. The point $(-1, 4)$ lies on the new line, so the equation of the new line is $y = 4$.

(f) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $3x + 2 = -3$, so $3x = \Box$, so $x = \Box$. The original line is vertical, so its gradient is undefined.
The original line is vertical, so the new line is horizontal. The point $(0, -4)$ lies on the new line, so the equation of the new line is $y = -4$.

4. (a) Let $(x_1, y_1) = (1, -4)$ and $(x_2, y_2) = (-3, -3)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so

$d = \sqrt{(1+3)^2 + (-4+3)^2} = \sqrt{4^2 + -1^2} = \sqrt{16+1} = \sqrt{17}$.

(b) $-2y - 3x = -2$, so $-2y = 3x - 2$, so $2y = -3x + 2$. Then dividing each side by 2 gives $y = \frac{-3}{2}x + \frac{2}{2} = -\frac{3}{2}x + 1$.
Hence the gradient is $m = -\frac{3}{2}$ and the y-intercept is $c = 1$.

(c) Thus the equation of the line is $y = -2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-3, 3)$ into this equation to get the value for c.

Hence $3 = -2 \times -3 + c$, so $3 = 6 + c$. Hence $c = 3 - 6 = -3$.

Hence the equation of the line is $y = -2x - 3$.

(d) Let $(x_1, y_1) = (1, -2)$ and $(x_2, y_2) = (3, 0)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0+2}{3-1} = \frac{2}{2} = 1$. Hence $m = 1$.

Thus the equation of the line is $y = x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (1, -2)$ into this equation to get the value for c.

Hence $-2 = 1 \times 1 + c$, so $-2 = 1 + c$. Hence $c = -2 - 1 = -3$.

Hence the equation of the line is $y = x - 3$.

(e) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $-2y + 2x - 2 = -1$, so $-2y = -2x + \Box$. Dividing each side by -2 gives $y = \frac{-2}{-2}x + \Box$, so $y = x + \Box$.
Hence the gradient of the original line is $m = 1$.

The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (2, 4)$ into this equation to get the value for c.

Hence $4 = 1 \times 2 + c$, so $4 = 2 + c$. Hence $c = 4 - 2 = 2$.

Hence the equation of the line is $y = x + 2$.

(f) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $-y-2=3$, so $-y=\Box$, so $y=\Box$. The original line is horizontal, so has gradient equal to 0.
The original line is horizontal, so the new line is vertical. The point $(0,3)$ lies on the new line, so the equation of the new line is $x=0$.

5. (a) Let $(x_1, y_1) = (4, -3)$ and $(x_2, y_2) = (-4, 3)$. Then $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$, so

$d = \sqrt{(4+4)^2 + (-3-3)^2} = \sqrt{8^2 + -6^2} = \sqrt{64+36} = \sqrt{100} = 10.$

(b) $-2y-2x+3=1$, so $-2y = 1+2x-3$, so $-2y = 2x-2$, so $2y = -2x+2$. Then dividing each side by 2 gives $y = \frac{-2}{2}x + \frac{2}{2} = -x+1$. Hence the gradient is $m=-1$ and the y-intercept is $c=1$.

(c) Thus the equation of the line is $y = x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-1, -1)$ into this equation to get the value for c.

Hence $-1 = 1 \times -1 + c$, so $-1 = -1 + c$. Hence $c = -1 + 1 = 0$.

Hence the equation of the line is $y = x$.

(d) Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (3, 2)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{3-2} = \frac{2}{1} = 2$. Hence $m = 2$.

Thus the equation of the line is $y = 2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (2, 0)$ into this equation to get the value for c.

Hence $0 = 2 \times 2 + c$, so $0 = 4 + c$. Hence $c = 0 - 4 = -4$.

Hence the equation of the line is $y = 2x - 4$.

(e) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $2y - 3x = 0$, so $2y = 3x + \Box$. Dividing each side by 2 gives $y = \frac{3}{2}x + \Box$. Hence the gradient of the original line is $m = \frac{3}{2}$.

The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = \frac{3}{2}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-4, 4)$ into this equation to get the value for c.

Hence $4 = \frac{3}{2} \times -4 + c$, so $4 = \frac{-12}{2} + c$, so $4 = -6 + c$. Hence $c = 4 + 6 = 10$.

Hence the equation of the line is $y = \frac{3}{2}x + 10$.

(f) To find the equation of the new line, we first need the gradient of the original line. We don't care about any constant term(s) in the original line, so to make the working easier we'll simply write $\Box$ instead of any constant term(s).

Now $x = -1$, so $x = \Box$. The original line is vertical, so its gradient is undefined.
The original line is vertical, so the new line is horizontal. The point $(-2, -4)$ lies on the new line, so the equation of the new line is $y = -4$.