MATH1040 Solutions to Practice Questions 6

1. (a) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -3x & + & 4y & = & 9 & (1) \\ 4x & + & 3y & = & 13 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by -4 and multiply equation (2) by -3, giving

$$\begin{array}{rcrcrl} 12x & - & 16y & = & -36 & (3) \\ -12x & - & 9y & = & -39 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$12x - 12x - 16y - 9y = -36 - 39. \quad (5)$$

Simplifying equation (5) gives

$$0 - 25y = -75. \quad (6)$$

Then solving equation (6) gives $\quad -25y = -75 \quad$ so $\quad y = \dfrac{-75}{-25} \quad = 3.$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$12x - 16 \times 3 = -36 \quad \text{so} \quad 12x = 12 \quad \text{so} \quad x = \frac{12}{12} \; = \; 1.$$

Hence the simultaneous solution to equations (1) and (2) is $x = 1$ and $y = 3$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -3x + 4y = -3 \times 1 + 4 \times 3 = -3 + 12 = 9$, as required.

(2) $\rightarrow 4x + 3y = 4 \times 1 + 3 \times 3 = 4 + 9 = 13$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -5x & + & 5y & = & -2 & (1) \\ 3x & - & y & = & -1 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (2) gives

$$y = 3x + 1 \quad (3)$$

Substituting equation (3) into equation (1) gives

$$5(3x + 1) - 5x = -2. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$15x + 5 - 5x = -2, \quad (5)$$

and simplifying equation (5) gives

$$10x + 5 = -2. \quad (6)$$

Then solving equation (6) gives $\quad 10x = -7 \quad$ so $\quad x = \dfrac{-7}{10} \quad = -\dfrac{7}{10}.$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = 3 \times \frac{-7}{10} + 1 \quad \text{so } y = \frac{-11}{10} \; = \; -\frac{11}{10}.$$

Hence the simultaneous solution to equations (1) and (2) is $x = -\dfrac{7}{10}$ and $y = -\dfrac{11}{10}$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -5x + 5y = -5 \times -\dfrac{7}{10} + 5 \times -\dfrac{11}{10} = \dfrac{35}{10} - \dfrac{55}{10} = -2$, as required.

(2) $\rightarrow 3x - y = 3 \times -\dfrac{7}{10} - 1 \times -\dfrac{11}{10} = \dfrac{-21}{10} + \dfrac{11}{10} = -1$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -2x & - & 2y & = & 5 & (1) \\ 2x & + & 2y & = & 5 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$-2x + 2x - 2y + 2y = 5 + 5. \quad (3)$$

Simplifying equation (3) gives

$$0 + 0 = 10. \quad (4)$$

Then solving equation (4) gives $\quad 0 = 10$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} x & - & 8y & = & 1 & (1) \\ -x & + & 8y & = & -1 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$x - x - 8y + 8y = 1 - 1. \quad (3)$$

Simplifying equation (3) gives

$$0 + 0 = 0. \quad (4)$$

Then solving equation (4) gives $\quad 0 = 0$.

This statement is **always** true, so there is an infinite number of solutions to the given equations.

2. (a) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -2x & - & y & = & 7 & (1) \\ -4x & + & 5y & = & -21 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (1) gives

$$y = -2x - 7 \quad (3)$$

Substituting equation (3) into equation (2) gives

$$5(-2x - 7) - 4x = -21. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$-10x - 35 - 4x = -21, \quad (5)$$

and simplifying equation (5) gives

$$-14x - 35 = -21. \quad (6)$$

Then solving equation (6) gives $\quad -14x = 14 \quad$ so $\quad x = \dfrac{14}{-14} \quad = -1.$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = -2 \times -1 - 7 \quad \text{so } y = -5.$$

Hence the simultaneous solution to equations (1) and (2) is $x = -1$ and $y = -5$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -2x - y = -2 \times -1 - 1 \times -5 = 2 + 5 = 7$, as required.

(2) $\rightarrow -4x + 5y = -4 \times -1 + 5 \times -5 = 4 - 25 = -21$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 3x & + & 2y & = & -4 & (1) \\ 3x & + & 4y & = & -4 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by -1, giving

$$\begin{array}{rcrcrl} -3x & - & 2y & = & 4 & (3) \\ 3x & + & 4y & = & -4 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$-3x + 3x - 2y + 4y = 4 - 4. \quad (5)$$

Simplifying equation (5) gives

$$0 + 2y = 0. \quad (6)$$

Then solving equation (6) gives $\quad 2y = 0 \quad$ so $\quad y = \dfrac{0}{2} \quad = 0.$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$-3x - 2 \times 0 = 4 \quad \text{so} \quad -3x = 4 \quad \text{so} \quad x = \frac{4}{-3} \quad = -\frac{4}{3}.$$

Hence the simultaneous solution to equations (1) and (2) is $x = -\dfrac{4}{3}$ and $y = 0$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow 3x + 2y = 3 \times -\dfrac{4}{3} + 2 \times 0 = \dfrac{-12}{3} + 0 = -4$, as required.

(2) $\rightarrow 3x + 4y = 3 \times -\dfrac{4}{3} + 4 \times 0 = \dfrac{-12}{3} + 0 = -4$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -4x & + & 8y & = & -7 & (1) \\ -x & + & 2y & = & 8 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (2) gives

$$x = 2y - 8 \quad (3)$$

Substituting equation (3) into equation (1) gives

$$-4(2y - 8) + 8y = -7. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$-8y + 32 + 8y = -7, \quad (5)$$

and simplifying equation (5) gives

$$0 + 32 = -7. \quad (6)$$

Then solving equation (6) gives $\quad 0 = -39$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -7x & - & y & = & 4 & (1) \\ 7x & + & y & = & -4 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$-7x + 7x - y + y = 4 - 4. \quad (3)$$

Simplifying equation (3) gives

$$0 + 0 = 0. \quad (4)$$

Then solving equation (4) gives $\quad 0 = 0$.

This statement is **always** true, so there is an infinite number of solutions to the given equations.

3. **(a)** First we number the equations for convenience.

$$\begin{array}{rcrcrl} -3x & - & y & = & -13 & (1) \\ -2x & - & 5y & = & -13 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (1) gives

$$y = -3x + 13 \quad (3)$$

Substituting equation (3) into equation (2) gives

$$-5(-3x + 13) - 2x = -13. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$15x - 65 - 2x = -13, \quad (5)$$

and simplifying equation (5) gives

$$13x - 65 = -13. \quad (6)$$

Then solving equation (6) gives $\quad 13x = 52 \quad$ so $\quad x = \dfrac{52}{13} \quad = \ 4.$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = -3 \times 4 + 13 \quad \text{so } y = 1.$$

Hence the simultaneous solution to equations (1) and (2) is $x = 4$ and $y = 1$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -3x - y = -3 \times 4 - 1 \times 1 = -12 - 1 = -13$, as required.

(2) $\rightarrow -2x - 5y = -2 \times 4 - 5 \times 1 = -8 - 5 = -13$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -3x & + & 3y & = & 5 & (1) \\ 5x & - & y & = & 3 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (2) gives

$$y = 5x - 3 \quad (3)$$

Substituting equation (3) into equation (1) gives

$$3(5x - 3) - 3x = 5. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$15x - 9 - 3x = 5, \quad (5)$$

and simplifying equation (5) gives

$$12x - 9 = 5. \quad (6)$$

Then solving equation (6) gives $\quad 12x = 14 \quad$ so $\quad x = \dfrac{14}{12} \quad = \dfrac{7}{6}.$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = 5 \times \frac{7}{6} - 3 \quad \text{so } y = \frac{17}{6}.$$

Hence the simultaneous solution to equations (1) and (2) is $x = \frac{7}{6}$ and $y = \frac{17}{6}$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -3x + 3y = -3 \times \frac{7}{6} + 3 \times \frac{17}{6} = \frac{-21}{6} + \frac{51}{6} = 5$, as required.

(2) $\rightarrow 5x - y = 5 \times \frac{7}{6} - 1 \times \frac{17}{6} = \frac{35}{6} - \frac{17}{6} = 3$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 3x & - & 3y & = & 1 & (1) \\ 8x & - & 8y & = & 3 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by 8 and multiply equation (2) by -3, giving

$$\begin{array}{rcrcrl} 24x & - & 24y & = & 8 & (3) \\ -24x & + & 24y & = & -9 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$24x - 24x - 24y + 24y = 8 - 9. \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = -1. \quad (6)$$

Then solving equation (6) gives $0 = -1$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 3x & + & 3y & = & 3 & (1) \\ 6x & + & 6y & = & 6 & (2) \end{array}$$

Next we take out any common factors.

$$\begin{array}{rcrcrl} x & + & y & = & 1 & (3) \\ x & + & y & = & 1 & (4) \end{array}$$

We solve these using elimination. Multiply equation (3) by -1, giving

$$\begin{array}{rcrcrl} -x & - & y & = & -1 & (5) \\ x & + & y & = & 1 & (6) \end{array}$$

Now we add both sides of equations (5) and (6), giving

$$-x + x - y + y = -1 + 1. \quad (7)$$

Simplifying equation (7) gives

$$0 + 0 = 0. \quad (8)$$

Then solving equation (8) gives $0 = 0$.

This statement is **always** true, so there is an infinite number of solutions to the given equations.

4. (a) First we number the equations for convenience.

$$\begin{array}{rcrcrl} x & + & 4y & = & 19 & (1) \\ -x & + & 4y & = & 13 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$x - x + 4y + 4y = 19 + 13. \quad (3)$$

Simplifying equation (3) gives

$$0 + 8y = 32. \quad (4)$$

Then solving equation (4) gives $\quad 8y = 32 \quad$ so $\quad y = \dfrac{32}{8} \quad = 4.$

Next we substitute the value for y into equation (1) to obtain the value for x, giving

$$x + 4 \times 4 = 19 \quad \text{so} \quad x = 3\ .$$

Hence the simultaneous solution to equations (1) and (2) is $x = 3$ and $y = 4$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\to x + 4y = 1 \times 3 + 4 \times 4 = 3 + 16 = 19$, as required.

(2) $\to -x + 4y = -1 \times 3 + 4 \times 4 = -3 + 16 = 13$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 5x & + & 2y & = & 5 & (1) \\ -5x & + & 5y & = & 1 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$5x - 5x + 2y + 5y = 5 + 1. \quad (3)$$

Simplifying equation (3) gives

$$0 + 7y = 6. \quad (4)$$

Then solving equation (4) gives $\quad 7y = 6 \quad$ so $\quad y = \dfrac{6}{7}\quad .$

Next we substitute the value for y into equation (1) to obtain the value for x, giving

$$5x + 2 \times \frac{6}{7} = 5 \quad \text{so} \quad 5x = \frac{23}{7} \quad \text{so} \quad x = \frac{23}{35}.$$

Hence the simultaneous solution to equations (1) and (2) is $x = \dfrac{23}{35}$ and $y = \dfrac{6}{7}$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\to 5x + 2y = 5 \times \dfrac{23}{35} + 2 \times \dfrac{6}{7} = \dfrac{115}{35} + \dfrac{12}{7} = 5$, as required.

(2) $\to -5x + 5y = -5 \times \dfrac{23}{35} + 5 \times \dfrac{6}{7} = \dfrac{-115}{35} + \dfrac{30}{7} = 1$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 3x & + & 3y & = & 7 & (1) \\ 7x & + & 7y & = & 2 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by -7 and multiply equation (2) by 3, giving

$$\begin{array}{rcrcrl} -21x & - & 21y & = & -49 & (3) \\ 21x & + & 21y & = & 6 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$-21x + 21x - 21y + 21y = -49 + 6. \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = -43. \quad (6)$$

Then solving equation (6) gives $\quad 0 = -43$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 2x & - & 4y & = & 8 & (1) \\ x & - & 2y & = & 4 & (2) \end{array}$$

Next we take out any common factors.

$$\begin{array}{rcrcrl} x & - & 2y & = & 4 & (3) \\ x & - & 2y & = & 4 & (4) \end{array}$$

We solve these using elimination. Multiply equation (3) by -1, giving

$$\begin{array}{rcrcrl} -x & + & 2y & = & -4 & (5) \\ x & - & 2y & = & 4 & (6) \end{array}$$

Now we add both sides of equations (5) and (6), giving

$$-x + x + 2y - 2y = -4 + 4. \quad (7)$$

Simplifying equation (7) gives

$$0 + 0 = 0. \quad (8)$$

Then solving equation (8) gives $\quad 0 = 0$.

This statement is **always** true, so there is an infinite number of solutions to the given equations.

5. **(a)** First we number the equations for convenience.

$$\begin{array}{rcrcrl} x & - & 4y & = & 7 & (1) \\ -3x & - & 2y & = & -7 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (1) gives

$$x = 4y + 7 \quad (3)$$

Substituting equation (3) into equation (2) gives

$$-3(4y + 7) - 2y = -7. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$-12y - 21 - 2y = -7, \quad (5)$$

and simplifying equation (5) gives

$$-14y - 21 = -7. \quad (6)$$

Then solving equation (6) gives $\quad -14y = 14 \quad$ so $\quad y = \dfrac{14}{-14} \quad = -1.$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = 4 \times -1 + 7 \quad \text{so } x = 3.$$

Hence the simultaneous solution to equations (1) and (2) is $x = 3$ and $y = -1$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow x - 4y = 1 \times 3 - 4 \times -1 = 3 + 4 = 7$, as required.

(2) $\rightarrow -3x - 2y = -3 \times 3 - 2 \times -1 = -9 + 2 = -7$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} x & - & y & = & 1 & (1) \\ -2x & + & 3y & = & 2 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (1) gives

$$x = y + 1 \quad (3)$$

Substituting equation (3) into equation (2) gives

$$-2(y+1)+3y=2. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$-2y-2+3y=2, \quad (5)$$

and simplifying equation (5) gives

$$y-2=2. \quad (6)$$

Then solving equation (6) gives $y = 4$.

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = 1 \times 4 + 1 \qquad \text{so } x = 5.$$

Hence the simultaneous solution to equations (1) and (2) is $x = 5$ and $y = 4$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow x - y = 1 \times 5 - 1 \times 4 = 5 - 4 = 1$, as required.

(2) $\rightarrow -2x + 3y = -2 \times 5 + 3 \times 4 = -10 + 12 = 2$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -5x & - & 5y & = & 4 & (1) \\ -5x & - & 5y & = & 7 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by -1, giving

$$\begin{array}{rcrcrl} 5x & + & 5y & = & -4 & (3) \\ -5x & - & 5y & = & 7 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$5x - 5x + 5y - 5y = -4 + 7. \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = 3. \quad (6)$$

Then solving equation (6) gives $0 = 3$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -4x & + & 4y & = & -4 & (1) \\ 8x & - & 8y & = & 8 & (2) \end{array}$$

Next we take out any common factors.

$$\begin{array}{rcrcrl} -x & + & y & = & -1 & (3) \\ x & - & y & = & 1 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$-x + x + y - y = -1 + 1. \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = 0. \quad (6)$$

Then solving equation (6) gives $0 = 0$.

This statement is **always** true, so there is an infinite number of solutions to the given equations.