1. **(a)** $f(-4) = -((-4)^2 + (-4)) = -(16 - 4) = -12.$

(b) $f(4) = 4^2 - 4 = 16 - 4 = 12.$

(c) $f(-4) = -((-4)^2 - (-4)) = -(16 + 4) = -20.$

(d) Square root only works on positive numbers (or 0), so the domain is $x \in [0, \infty)$.

(e) It is not possible to have 0 as the denominator of a fraction. This happens when $\mid -11 + x \mid = 0$, so $x = 11$. Hence the domain is **everything else**, so $x \in (-\infty, 11) \cup (11, \infty)$.

(f) Square root only works on positive numbers (or 0), so the domain is $x \in [0, \infty)$.

(g) The numerator is negative and the denominator is positive, so it is not possible to get a positive value for y. Also, it is not possible to have $y = 0$. Hence the range is $y \in (-\infty, 0)$.

(h) Both denominator and numerator are positive, so it is not possible to get a negative value for y. Also, it is not possible to have $y = 0$. Hence the range is $y \in (0, \infty)$.

(i) Square root is always positive (or 0), so the range is $y \in [9, \infty)$.

(j) To solve each of these, remember that if $a \times b = 0$ then either $a = 0$ or $b = 0$. Then:

(i) $3x(-x + 2) = 0.$ Hence (1) $3x = 0$ or (2) $(-x + 2) = 0$. Solve these:

(1) $x = 0.$
(2) $-x = -2$, so $x = 2$.
Hence the solutions are $x = 0$ and $x = 2$.

(ii) $(-4x - 2)(-3x - 1) = 0.$ Hence (1) $-4x - 2 = 0$ or (2) $-3x - 1 = 0$. Solve these:

(1) $-4x = 2$, so $x = \frac{2}{-4}$, so $x = -\frac{1}{2}$.

(2) $-3x = 1$, so $x = \frac{1}{-3}$, so $x = -\frac{1}{3}$.
Hence the solutions are $x = -\frac{1}{2}$ and $x = -\frac{1}{3}$.

(iii) $-3(-3x + 3)(x - 2) = 0.$ Hence (1) $-3x + 3 = 0$ or (2) $x - 2 = 0$. Solve these:

(1) $-3x = -3$, so $x = \frac{-3}{-3}$, so $x = 1$.

(2) $x = 2$.
Hence the solutions are $x = 1$ and $x = \frac{2}{1}$.

(iv) $(-4x + 2)^2 = 0.$ Hence $-4x + 2 = 0$, so $-4x = -2$, so $x = \frac{-2}{-4}$, so $x = \frac{1}{2}$.

(k) $-3x^2 + 2x - 1 = 0$, so we use $a = -3$, $b = 2$ and $c = -1$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - (4 \times -3 \times -1)}}{2 \times -3} \\ &= \frac{-2 \pm \sqrt{4 - 12}}{-6} \\ &= \frac{-2 \pm \sqrt{-8}}{-6}. \end{aligned}$$

Hence there is no solution.

(l) $4x^2 - 7x = 0$, so we use $a = 4$, $b = -7$ and $c = 0$ in the quadratic formula. Hence

$$\begin{aligned}
x &= \frac{7 \pm \sqrt{(-7)^2 - (4 \times 4 \times 0)}}{2 \times 4} \\
&= \frac{7 \pm \sqrt{49 - 0}}{8} \\
&= \frac{7 \pm \sqrt{49}}{8} \\
&= \frac{7 \pm 7}{8} \\
&= \frac{7+7}{8} \text{ or } \frac{7-7}{8} \\
&= \frac{14}{8} \text{ or } \frac{0}{8} \\
&= \frac{7}{4} \text{ or } 0.
\end{aligned}$$

2. **(a)** $f(2) = -(2^2) + 2 = -4 + 2 = -2$.

(b) $f(-2) = -((-2)^2 - (-2)) = -(4 + 2) = -6$.

(c) $f(1) = 1^2 - 1 = 1 - 1 = 0$.

(d) It is not possible to have 0 as the denominator of a fraction. This happens when $-5x + 1 = 0$, so $x = \frac{-1}{-5}$, so $x = \frac{1}{5}$. Hence the domain is **everything else**, so $x \in (-\infty, \frac{1}{5}) \cup (\frac{1}{5}, \infty)$.

(e) Square root only works on positive numbers (or 0), so the domain is $x \in (-\infty, 0]$.

(f) Square root only works on positive numbers and the denominator of a fraction cannot be 0, so the domain is $x \in (0, \infty)$.

(g) The only value of y which is not possible is $y = 0$. Hence the range is **everything else**, so $y \in (-\infty, 0) \cup (0, \infty)$.

(h) The numerator is negative and the denominator is positive, so it is not possible to get a positive value for y. Also, it is not possible to have $y = 0$. Hence the range is $y \in (-\infty, 0)$.

(i) The only value of y which is not possible is $y = 0$. Hence the range is **everything else**, so $y \in (-\infty, 0) \cup (0, \infty)$.

(j) To solve each of these, remember that if $a \times b = 0$ then either $a = 0$ or $b = 0$. Then:

(i) $4x(3x + 2) = 0$. Hence (1) $4x = 0$ or (2) $(3x + 2) = 0$. Solve these:

(1) $x = 0$.

(2) $3x = -2$, so $x = \frac{-2}{3}$, so $x = -\frac{2}{3}$.

Hence the solutions are $x = 0$ and $x = -\frac{2}{3}$.

(ii) $(2x - 1)(3x - 3) = 0$. Hence (1) $2x - 1 = 0$ or (2) $3x - 3 = 0$. Solve these:

(1) $2x = 1$, so $x = \frac{1}{2}$.

(2) $3x = 3$, so $x = \frac{3}{3}$, so $x = 1$.

Hence the solutions are $x = \frac{1}{2}$ and $x = 1$.

(iii) $-2(3x + 1)(4x - 3) = 0$. Hence (1) $3x + 1 = 0$ or (2) $4x - 3 = 0$. Solve these:

(1) $3x = -1$, so $x = \frac{-1}{3}$, so $x = -\frac{1}{3}$.

(2) $4x = 3$, so $x = \frac{3}{4}$.

Hence the solutions are $x = -\frac{1}{3}$ and $x = \frac{3}{4}$.

(iv) $(x-2)^2 = 0.$ Hence $x - 2 = 0,$ so $x = 2.$

(k) $-3x^2 + 4x - 2 = 0,$ so we use $a = -3$, $b = 4$ and $c = -2$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - (4 \times -3 \times -2)}}{2 \times -3} \\ &= \frac{-4 \pm \sqrt{16 - 24}}{-6} \\ &= \frac{-4 \pm \sqrt{-8}}{-6}. \end{aligned}$$

Hence there is no solution.

(l) $3x^2 + 8x = 0,$ so we use $a = 3$, $b = 8$ and $c = 0$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-8 \pm \sqrt{8^2 - (4 \times 3 \times 0)}}{2 \times 3} \\ &= \frac{-8 \pm \sqrt{64 - 0}}{6} \\ &= \frac{-8 \pm \sqrt{64}}{6} \\ &= \frac{-8 \pm 8}{6} \\ &= \frac{-8 + 8}{6} \text{ or } \frac{-8 - 8}{6} \\ &= \frac{0}{6} \text{ or } \frac{-16}{6} \\ &= 0 \text{ or } -\frac{8}{3}. \end{aligned}$$

3. **(a)** $f(-3) = (-3)^2 - (-3) = 9 + 3 = 12.$

(b) $f(-4) = -(-4)^2 + (-4) = -16 - 4 = -20.$

(c) $f(-1) = (-1)^2 + (-1) = 1 - 1 = 0.$

(d) Square root only works on positive numbers and the denominator of a fraction cannot be 0, so the domain is $x \in (0, \infty)$.

(e) It is not possible to have 0 as the denominator of a fraction. This happens when $\mid -7 + x \mid = 0$, so $x = 7$. Hence the domain is **everything else**, so $x \in (-\infty, 7) \cup (7, \infty)$.

(f) Square root only works on positive numbers (or 0), so the domain is $x \in [-12, \infty)$.

(g) The numerator is negative and the denominator is positive, so it is not possible to get a positive value for y. Also, it is not possible to have $y = 0$. Hence the range is $y \in (-\infty, 0)$.

(h) The square of every number is positive, so the range is $y \in [0, \infty)$.

(i) The only value of y which is not possible is $y = 0$. Hence the range is **everything else**, so $y \in (-\infty, 0) \cup (0, \infty)$.

(j) To solve each of these, remember that if $a \times b = 0$ then either $a = 0$ or $b = 0$. Then:

(i) $x(3x + 1) = 0.$ Hence (1) $x = 0$ or (2) $(3x + 1) = 0$. Solve these:

(1) $x = 0.$

(2) $3x = -1$, so $x = \dfrac{-1}{3}$, so $x = -\dfrac{1}{3}$.

Hence the solutions are $x = 0$ and $x = -\dfrac{1}{3}$.

(ii) $(-3x - 2)(x - 3) = 0.$ Hence (1) $-3x - 2 = 0$ or (2) $x - 3 = 0$. Solve these:

(1) $-3x = 2$, so $x = \dfrac{2}{-3}$, so $x = -\dfrac{2}{3}$.

(2) $x = 3.$

Hence the solutions are $x = -\dfrac{2}{3}$ and $x = \dfrac{3}{1}$.

(iii) $4(4x+3)(-x+3)=0.$ Hence (1) $4x+3=0$ or (2) $-x+3=0$. Solve these:

(1) $4x=-3$, so $x=\dfrac{-3}{4}$, so $x=-\dfrac{3}{4}$.

(2) $-x=-3$, so $x=3$.

Hence the solutions are $x=-\dfrac{3}{4}$ and $x=3$.

(iv) $(-4x+1)^2=0.$ Hence $-4x+1=0$, so $-4x=-1$, so $x=\dfrac{-1}{-4}$, so $x=\dfrac{1}{4}$.

(k) $4x^2-5x+4=0$, so we use $a=4$, $b=-5$ and $c=4$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{5\pm\sqrt{(-5)^2-(4\times4\times4)}}{2\times4} \\ &= \frac{5\pm\sqrt{25-64}}{8} \\ &= \frac{5\pm\sqrt{-39}}{8}. \end{aligned}$$

Hence there is no solution.

(l) $-4x^2+7x-3=0$, so we use $a=-4$, $b=7$ and $c=-3$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-7\pm\sqrt{7^2-(4\times-4\times-3)}}{2\times-4} \\ &= \frac{-7\pm\sqrt{49-48}}{-8} \\ &= \frac{-7\pm\sqrt{1}}{-8} \\ &= \frac{-7\pm1}{-8} \\ &= \frac{-7+1}{-8} \text{ or } \frac{-7-1}{-8} \\ &= \frac{-6}{-8} \text{ or } \frac{-8}{-8} \\ &= \frac{3}{4} \text{ or } 1. \end{aligned}$$

4. (a) $f(1)=1^2+1=1+1=2$.

(b) $f(4)=(-4)^2+4=16+4=20$.

(c) $f(1)=-(1^2)+1=-1+1=0$.

(d) Any value of x can be substituted into f, so the domain is $x\in(-\infty,\infty)$.

(e) Square root only works on positive numbers and the denominator of a fraction cannot be 0, so the domain is $x\in(0,\infty)$.

(f) Any value of x can be substituted into f, so the domain is $x\in(-\infty,\infty)$.

(g) The numerator is negative and the denominator is positive, so it is not possible to get a positive value for y. Also, it is not possible to have $y=0$. Hence the range is $y\in(-\infty,0)$.

(h) $x^2\geq0$ so $-2x^2\leq0$, so $-2x^2+8\leq8$, so the range is $(-\infty,8]$.

(i) Square root is always positive (or 0), so the range is $y\in[11,\infty)$.

(j) To solve each of these, remember that if $a\times b=0$ then either $a=0$ or $b=0$. Then:

(i) $4x(-2x-1)=0.$ Hence (1) $4x=0$ or (2) $(-2x-1)=0$. Solve these:

(1) $x=0$.

(2) $-2x=1$, so $x=\dfrac{1}{-2}$, so $x=-\dfrac{1}{2}$.

Hence the solutions are $x=0$ and $x=-\dfrac{1}{2}$.

(ii) $(-2x-3)(3x+2)=0$. Hence (1) $-2x-3=0$ or (2) $3x+2=0$. Solve these:

(1) $-2x=3$, so $x=\dfrac{3}{-2}$, so $x=-\dfrac{3}{2}$.

(2) $3x=-2$, so $x=\dfrac{-2}{3}$, so $x=-\dfrac{2}{3}$.

Hence the solutions are $x=-\dfrac{3}{2}$ and $x=-\dfrac{2}{3}$.

(iii) $3(-x+3)(2x-2)=0$. Hence (1) $-x+3=0$ or (2) $2x-2=0$. Solve these:

(1) $-x=-3$, so $x=3$.

(2) $2x=2$, so $x=\dfrac{2}{2}$, so $x=1$.

Hence the solutions are $x=3$ and $x=1$.

(iv) $(x+2)^2=0$. Hence $x+2=0$, so $x=-2$.

(k) $2x^2-3x+2=0$, so we use $a=2$, $b=-3$ and $c=2$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{3\pm\sqrt{(-3)^2-(4\times 2\times 2)}}{2\times 2} \\ &= \frac{3\pm\sqrt{9-16}}{4} \\ &= \frac{3\pm\sqrt{-7}}{4}. \end{aligned}$$

Hence there is no solution.

(l) $-2x^2-6x-4=0$, so we use $a=-2$, $b=-6$ and $c=-4$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{6\pm\sqrt{(-6)^2-(4\times -2\times -4)}}{2\times -2} \\ &= \frac{6\pm\sqrt{36-32}}{-4} \\ &= \frac{6\pm\sqrt{4}}{-4} \\ &= \frac{6\pm 2}{-4} \\ &= \frac{6+2}{-4} \text{ or } \frac{6-2}{-4} \\ &= \frac{8}{-4} \text{ or } \frac{4}{-4} \\ &= -2 \text{ or } -1. \end{aligned}$$

5. **(a)** $f(-3)=(-(-3))^2+(-3)=9-3=6$.

(b) $f(4)=-(4^2)+4=-16+4=-12$.

(c) $f(-1)=-(-1)^2-(-1)=-1+1=0$.

(d) Any value of x can be substituted into f, so the domain is $x\in(-\infty,\infty)$.

(e) Square root only works on positive numbers and the denominator of a fraction cannot be 0, so the domain is $x\in(14,\infty)$.

(f) It is not possible to have 0 as the denominator of a fraction. This happens when $x-5=0$, so $x=5$. Hence the domain is **everything else**, so $x\in(-\infty,5)\cup(5,\infty)$.

(g) Square root is always positive (or 0), so the range is $y\in[-12,\infty)$.

(h) $\sqrt{x}$ is always positive (or 0), so $f(x)=-12\sqrt{x}$ is always negative (or 0), so the range is $y\in(-\infty,0]$.

(i) $x^2 \geq 0$ so $2x^2 \geq 0$, so $2x^2 + 2 \leq 2$, so the range is $[2, \infty)$.

(j) To solve each of these, remember that if $a \times b = 0$ then either $a = 0$ or $b = 0$. Then:

(i) $4x(x-2) = 0$. Hence (1) $4x = 0$ or (2) $(x-2) = 0$. Solve these:

(1) $x = 0$.
(2) $x = 2$.

Hence the solutions are $x = 0$ and $x = \frac{2}{1}$.

(ii) $(-3x-3)(2x+2) = 0$. Hence (1) $-3x-3 = 0$ or (2) $2x+2 = 0$. Solve these:

(1) $-3x = 3$, so $x = \frac{3}{-3}$, so $x = -1$.

(2) $2x = -2$, so $x = \frac{-2}{2}$, so $x = -1$.

Hence the solutions are $x = -1$ and $x = -1$.

(iii) $4(4x-3)(-x-2) = 0$. Hence (1) $4x-3 = 0$ or (2) $-x-2 = 0$. Solve these:

(1) $4x = 3$, so $x = \frac{3}{4}$.

(2) $-x = 2$, so $x = -2$.

Hence the solutions are $x = \frac{3}{4}$ and $x = -2$.

(iv) $(-2x+1)^2 = 0$. Hence $-2x+1 = 0$, so $-2x = -1$, so $x = \frac{-1}{-2}$, so $x = \frac{1}{2}$.

(k) $x^2 + x + 3 = 0$, so we use $a = 1$, $b = 1$ and $c = 3$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1^2 - (4 \times 1 \times 3)}}{2 \times 1} \\ &= \frac{-1 \pm \sqrt{1 - 12}}{2} \\ &= \frac{-1 \pm \sqrt{-11}}{2}. \end{aligned}$$

Hence there is no solution.

(l) $4x^2 + 5x + 1 = 0$, so we use $a = 4$, $b = 5$ and $c = 1$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - (4 \times 4 \times 1)}}{2 \times 4} \\ &= \frac{-5 \pm \sqrt{25 - 16}}{8} \\ &= \frac{-5 \pm \sqrt{9}}{8} \\ &= \frac{-5 \pm 3}{8} \\ &= \frac{-5+3}{8} \text{ or } \frac{-5-3}{8} \\ &= \frac{-2}{8} \text{ or } \frac{-8}{8} \\ &= -\frac{1}{4} \text{ or } -1. \end{aligned}$$