MATH1040 Solutions to Practice Questions 8

1. (a) (i) $-y + 3x + 1 = 2$, so $-y = 2 - 3x - 1$, so $-y = -3x + 1$, so $y = 3x - 1$. Hence this is a straight line, with positive gradient and negative y-intercept. Hence the matching graph is Graph E.

(ii) $-3y - 2x - 3 = -2$, so $-3y = -2 + 2x + 3$, so $-3y = 2x + 1$, so $3y = -2x - 1$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

(iii) $3y - 2 = 1$, so $3y = 3$. Hence this is a horizontal line, with y positive. Hence the matching graph is Graph C.

(iv) $-y + 2x + 2 = 1$, so $-y = 1 - 2x - 2$, so $-y = -2x - 1$, so $y = 2x + 1$. Hence this is a straight line, with positive gradient and positive y-intercept. Hence the matching graph is Graph G.

(v) $y = -3x^2 - 2$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. The y-intercept is negative. Hence the matching graph is Graph T.

(vi) $y = e^{-3x}$, which is a graph of exponential decay. Hence the matching graph is Graph L.

(vii) $y = 3x^2$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is 0. Hence the matching graph is Graph P.

(viii) $y = e^x$, which is a graph of exponential growth. Hence the matching graph is Graph K.

(b) Let P be the amount invested, r be the interest rate per time period, x be the number of time periods and F be the final value. In each case, $P = 100$. Then:

(i) Interest compounds annually so we use the rate and number of time periods given in the question.
Hence $r = 12\% = 0.12$ and $n = 4$, so $F = 100(1 + 0.12)^4 = 100(1.12)^4 = 157.35$.
The final balance is \$157.35.

(ii) Interest compounds twice a year so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 6\% = 0.06$ and $n = 8$, so $F = 100(1 + 0.06)^8 = 100(1.06)^8 = 159.38$.
The final balance is \$159.38.

(iii) Interest compounds 12 times a year so we need to divide the given rate by 12 and multiply the given number of time periods by 12.
Hence $r = 1.0\% = 0.010$ and $n = 48$, so $F = 100(1 + 0.010)^{48} = 100(1.010)^{48} = 161.22$.
The final balance is \$161.22.

(iv) Interest compounds continuously, so $F = 100e^{0.12\times 4} = 100e^{0.48} = 161.61$.
The final balance is \$161.61.

(c) Given an angle a in radians, to convert a to degrees you multiply by 180 and divide by π. Hence the converted angles are:

$$360^\circ \quad -180^\circ \quad 270^\circ \quad -450^\circ \quad -60^\circ \quad -495^\circ \quad 390^\circ \quad 440^\circ.$$

(d) Given an angle a in degrees, to convert a to radians you divide by 180 and multiply by π. Hence the converted angles are:

$$\pi \quad -2\pi \quad \frac{5\pi}{2} \quad \frac{-3\pi}{2} \quad \frac{-2\pi}{3} \quad \frac{7\pi}{4} \quad \frac{\pi}{6} \quad \frac{16\pi}{9}.$$

(e) (i) $1 = 10^0$, so the answer is 0.

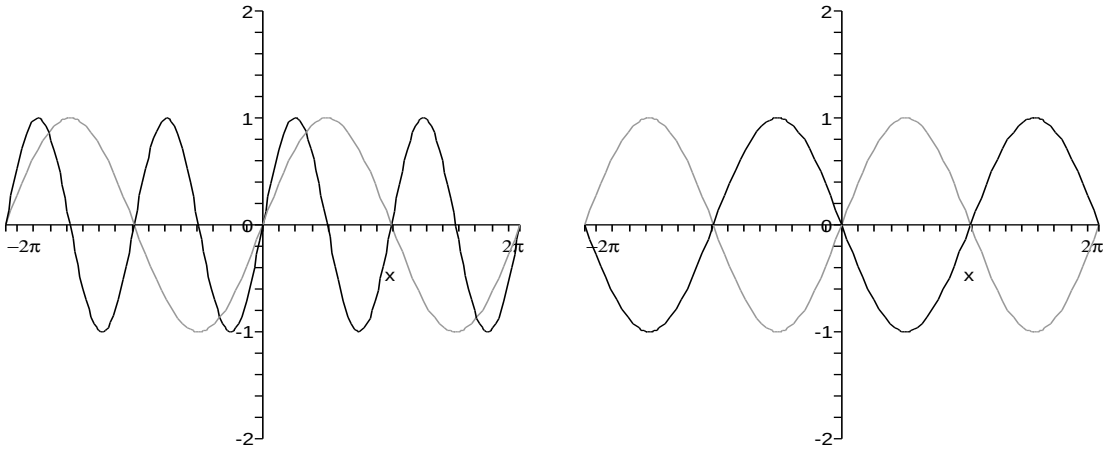
(ii) $\frac{1}{100000} = 10^{-5}$, so the answer is -5.

(iii) The answer is 4.

(iv) $\frac{1}{e^5} = e^{-5}$, so the answer is -5.

(f) The graph of $y_1 = \sin 2x$ is on the left, and the graph of $y_2 = -\sin x$ is on the right. In each case, the graph

of $\sin x$ is shaded more lightly.



2. **(a)** **(i)** $y = -3x^2 + 2$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is positive. Hence the matching graph is Graph R.

(ii) $2y + 2x + 1 = 2$, so $2y = 2 - 2x - 1$, so $2y = -2x + 1$. Hence this is a straight line, with negative gradient and positive y-intercept. Hence the matching graph is Graph H.

(iii) $y = 3 \mid 4x \mid$, which is a graph of absolute value. Hence the matching graph is Graph N.

(iv) $y = 2x^2 - 2$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. The y-intercept is negative. Hence the matching graph is Graph Q.

(v) $y = 3x^2$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is 0. Hence the matching graph is Graph P.

(vi) $y = -2x^2$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is 0. Hence the matching graph is Graph S.

(vii) $-y + 2x = 3$, so $-y = -2x + 3$, so $y = 2x - 3$. Hence this is a straight line, with positive gradient and negative y-intercept. Hence the matching graph is Graph E.

(viii) $3y - 3x - 3 = 3$, so $3y = 3 + 3x + 3$, so $3y = 3x + 6$. Hence this is a straight line, with positive gradient and positive y-intercept. Hence the matching graph is Graph G.

(b) Let P be the amount invested, r be the interest rate per time period, x be the number of time periods and F be the final value. In each case, $P = 100$. Then:

(i) Interest compounds annually so we use the rate and number of time periods given in the question.
Hence $r = 12\% = 0.12$ and $n = 8$, so $F = 100(1 + 0.12)^8 = 100(1.12)^8 = 247.60$.
The final balance is \$247.60.

(ii) Interest compounds twice a year so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 6\% = 0.06$ and $n = 16$, so $F = 100(1 + 0.06)^{16} = 100(1.06)^{16} = 254.03$.
The final balance is \$254.03.

(iii) Interest compounds 12 times a year so we need to divide the given rate by 12 and multiply the given number of time periods by 12.
Hence $r = 1.0\% = 0.010$ and $n = 96$, so $F = 100(1 + 0.010)^{96} = 100(1.010)^{96} = 259.93$.
The final balance is \$259.93.

(iv) Interest compounds continuously, so $F = 100e^{0.12 \times 8} = 100e^{0.96} = 261.17$.
The final balance is \$261.17.

(c) Given an angle a in radians, to convert a to degrees you multiply by 180 and divide by π. Hence the converted angles are:

$$360^\circ \qquad -900^\circ \qquad 90^\circ \qquad -450^\circ \qquad 420^\circ \qquad 135^\circ \qquad -510^\circ \qquad 380^\circ.$$

(d) Given an angle a in degrees, to convert a to radians you divide by 180 and multiply by π. Hence the converted angles are:

$$5\pi \qquad -2\pi \qquad \frac{\pi}{2} \qquad \frac{-3\pi}{2} \qquad \frac{2\pi}{3} \qquad \frac{-9\pi}{4} \qquad \frac{-13\pi}{6} \qquad \frac{-16\pi}{9}.$$

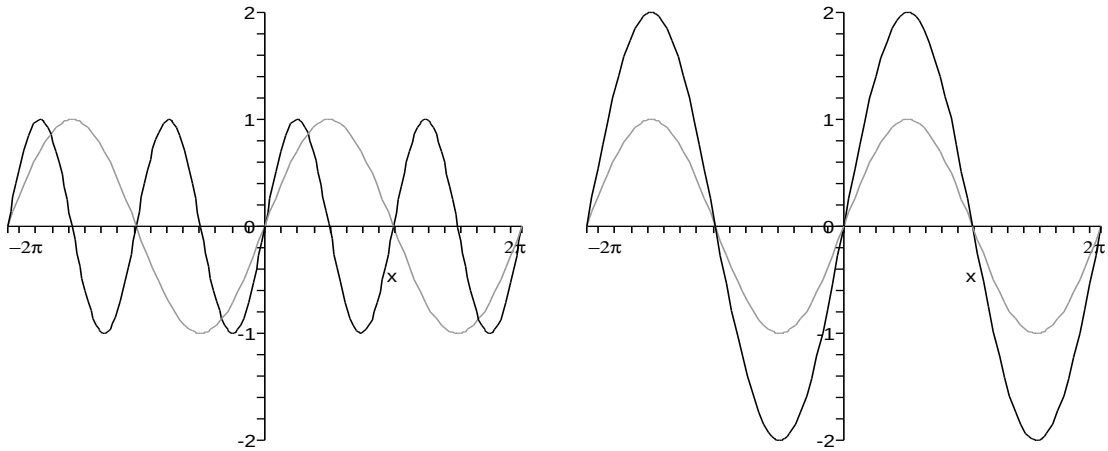
(e) **(i)** $100 = 10^2$, so the answer is 2.

(ii) $1 = 10^{-0}$, so the answer is 0.

(iii) The answer is 6.

(iv) $1 = e^0$, so the answer is 0.

(f) The graph of $y_1 = \sin 2x$ is on the left, and the graph of $y_2 = 2\sin x$ is on the right. In each case, the graph of $\sin x$ is shaded more lightly.



3. **(a)** **(i)** $y = -2x^2 - 1$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. The y-intercept is negative. Hence the matching graph is Graph T.

(ii) $y + 2x - 1 = 2$, so $y = 2 - 2x + 1$, so $y = -2x + 3$. Hence this is a straight line, with negative gradient and positive y-intercept. Hence the matching graph is Graph H.

(iii) $-2y - 2x = 1$, so $-2y = 2x + 1$, so $2y = -2x - 1$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

(iv) $y = x^2 + 1$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is positive. Hence the matching graph is Graph O.

(v) $y = e^{-x}$, which is a graph of exponential decay. Hence the matching graph is Graph L.

(vi) $-3y + x - 1 = 3$, so $-3y = 3 - x + 1$, so $-3y = -x + 4$, so $3y = x - 4$. Hence this is a straight line, with positive gradient and negative y-intercept. Hence the matching graph is Graph E.

(vii) $-y + x - 1 = -3$, so $-y = -3 - x + 1$, so $-y = -x - 2$, so $y = x + 2$. Hence this is a straight line, with positive gradient and positive y-intercept. Hence the matching graph is Graph G.

(viii) $y = -2 \mid 4x \mid$, which is a graph of negative absolute value. Hence the matching graph is Graph M.

(b) Let P be the amount invested, r be the interest rate per time period, x be the number of time periods and F be the final value. In each case, $P = 100$. Then:

(i) Interest compounds annually so we use the rate and number of time periods given in the question.
Hence $r = 24\% = 0.24$ and $n = 6$, so $F = 100(1 + 0.24)^6 = 100(1.24)^6 = 363.52$.
The final balance is \$363.52.

(ii) Interest compounds twice a year so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 12\% = 0.12$ and $n = 12$, so $F = 100(1 + 0.12)^{12} = 100(1.12)^{12} = 389.60$.
The final balance is \$389.60.

(iii) Interest compounds 12 times a year so we need to divide the given rate by 12 and multiply the given number of time periods by 12.
Hence $r = 2.0\% = 0.020$ and $n = 72$, so $F = 100(1 + 0.020)^{72} = 100(1.020)^{72} = 416.11$.
The final balance is \$416.11.

(iv) Interest compounds continuously, so $F = 100e^{0.24\times 6} = 100e^{1.44} = 422.07$.
The final balance is \$422.07.

(c) Given an angle a in radians, to convert a to degrees you multiply by 180 and divide by π. Hence the converted angles are:

$$540^\circ \quad -360^\circ \quad 450^\circ \quad -90^\circ \quad -480^\circ \quad -315^\circ \quad -150^\circ \quad 340^\circ.$$

(d) Given an angle a in degrees, to convert a to radians you divide by 180 and multiply by π. Hence the converted angles are:

$$2\pi \qquad -4\pi \qquad \frac{5\pi}{2} \qquad \frac{-\pi}{2} \qquad \frac{-2\pi}{3} \qquad \frac{-3\pi}{4} \qquad \frac{17\pi}{6} \qquad \frac{10\pi}{9}.$$

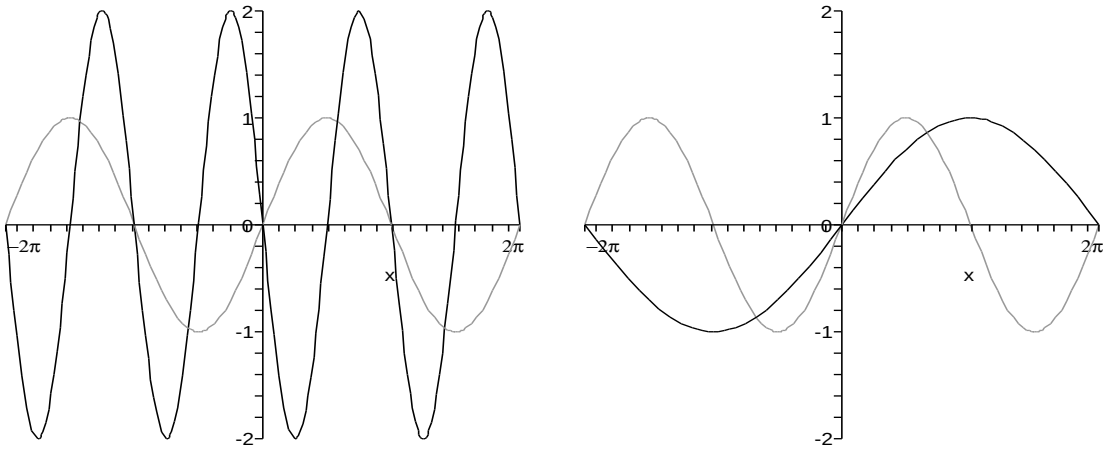
(e) **(i)** $1000 = 10^3$, so the answer is 3.

(ii) $\dfrac{1}{1000000} = 10^{-6}$, so the answer is -6.

(iii) $e^0 = 1$, so the answer is 0.

(iv) $\dfrac{1}{e^1} = e^{-1}$, so the answer is -1.

(f) The graph of $y_1 = -2\sin 2x$ is on the left, and the graph of $y_2 = \sin\dfrac{x}{2}$ is on the right. In each case, the graph of $\sin x$ is shaded more lightly.



4. **(a)** **(i)** $3y - 2x + 1 = 3$, so $3y = 3 + 2x - 1$, so $3y = 2x + 2$. Hence this is a straight line, with positive gradient and positive y-intercept. Hence the matching graph is Graph G.

(ii) $-2y + 3x - 1 = 3$, so $-2y = 3 - 3x + 1$, so $-2y = -3x + 4$, so $2y = 3x - 4$. Hence this is a straight line, with positive gradient and negative y-intercept. Hence the matching graph is Graph E.

(iii) $y = -2x^2 + 1$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is positive. Hence the matching graph is Graph R.

(iv) $x - 3 = 0$, so $x = 3$. Hence this is a vertical line, with x positive. Hence the matching graph is Graph B.

(v) $y = -3 \mid -3x \mid$, so $y = -3 \mid 3x \mid$, which is a graph of negative absolute value. Hence the matching graph is Graph M.

(vi) $y + 2x = -1$, so $y = -2x - 1$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

(vii) $-3y - 3x - 2 = -2$, so $-3y = -2 + 3x + 2$, so $-3y = 3x$, so $3y = -3x$. Hence this is a straight line, with negative gradient and passing through the origin. Hence the matching graph is Graph I.

(viii) $y = 2x^2 + 1$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is positive. Hence the matching graph is Graph O.

(b) Let P be the amount invested, r be the interest rate per time period, x be the number of time periods and F be the final value. In each case, $P = 100$. Then:

(i) Interest compounds annually so we use the rate and number of time periods given in the question.
Hence $r = 30\% = 0.30$ and $n = 6$, so $F = 100(1 + 0.30)^6 = 100(1.30)^6 = 482.68$.
The final balance is \$482.68.

(ii) Interest compounds twice a year so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 15\% = 0.15$ and $n = 12$, so $F = 100(1 + 0.15)^{12} = 100(1.15)^{12} = 535.02$.
The final balance is \$535.02.

(iii) Interest compounds 12 times a year so we need to divide the given rate by 12 and multiply the given number of time periods by 12.
Hence $r = 2.5\% = 0.025$ and $n = 72$, so $F = 100(1 + 0.025)^{72} = 100(1.025)^{72} = 591.72$.
The final balance is \$591.72.

(iv) Interest compounds continuously, so $F = 100e^{0.30\times 6} = 100e^{1.80} = 604.96$.
The final balance is \$604.96.

(c) Given an angle a in radians, to convert a to degrees you multiply by 180 and divide by π. Hence the converted angles are:

$$540^\circ \qquad -360^\circ \qquad 450^\circ \qquad -270^\circ \qquad 480^\circ \qquad -135^\circ \qquad 210^\circ \qquad 320^\circ.$$

(d) Given an angle a in degrees, to convert a to radians you divide by 180 and multiply by π. Hence the converted angles are:

$$2\pi \qquad -3\pi \qquad \frac{5\pi}{2} \qquad \frac{-3\pi}{2} \qquad \frac{-2\pi}{3} \qquad \frac{-3\pi}{4} \qquad \frac{-7\pi}{6} \qquad \frac{22\pi}{9}.$$

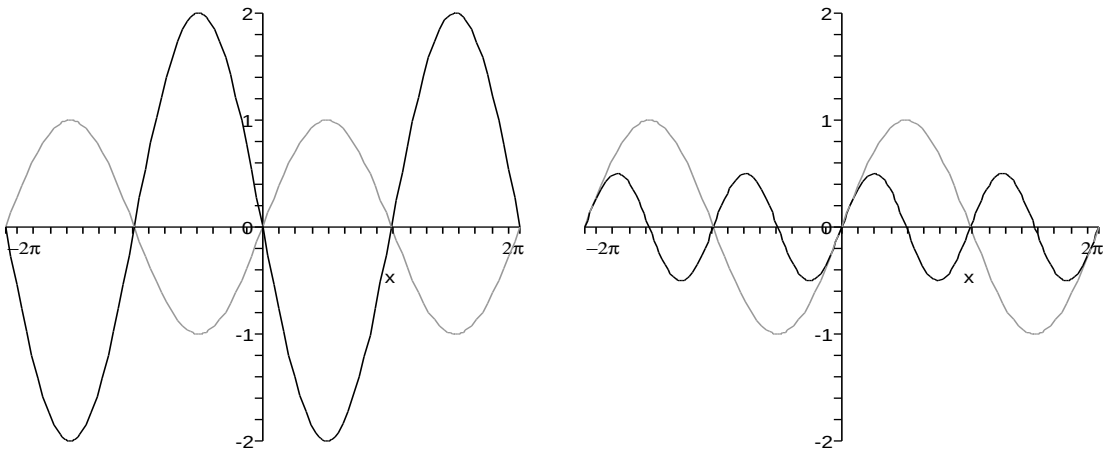
(e) **(i)** $1000000 = 10^6$, so the answer is 6.

(ii) $\dfrac{1}{10000} = 10^{-4}$, so the answer is -4.

(iii) $e^0 = 1$, so the answer is 0.

(iv) $\dfrac{1}{e^1} = e^{-1}$, so the answer is -1.

(f) The graph of $y_1 = -2\sin x$ is on the left, and the graph of $y_2 = \dfrac{1}{2}\sin 2x$ is on the right. In each case, the graph of $\sin x$ is shaded more lightly.



5. **(a)** **(i)** $y + 2x - 1 = -2$, so $y = -2 - 2x + 1$, so $y = -2x - 1$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

(ii) $y = -3 \mid -4x \mid$, so $y = -3 \mid 4x \mid$, which is a graph of negative absolute value. Hence the matching graph is Graph M.

(iii) $y = 4 \mid -3x \mid$, so $y = 4 \mid 3x \mid$, which is a graph of absolute value. Hence the matching graph is Graph N.

(iv) $3y - 3x + 2 = 0$, so $3y = 0 + 3x - 2$, so $3y = 3x - 2$. Hence this is a straight line, with positive gradient and negative y-intercept. Hence the matching graph is Graph E.

(v) $2y + 2x - 2 = -2$, so $2y = -2 - 2x + 2$, so $2y = -2x$. Hence this is a straight line, with negative gradient and passing through the origin. Hence the matching graph is Graph I.

(vi) $y = 3x^2 - 2$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. The y-intercept is negative. Hence the matching graph is Graph Q.

(vii) $-2y - x + 2 = -1$, so $-2y = -1 + x - 2$, so $-2y = x - 3$, so $2y = -x + 3$. Hence this is a straight line, with negative gradient and positive y-intercept. Hence the matching graph is Graph H.

(viii) $x - 3 = 3$, so $x = 6$. Hence this is a vertical line, with x positive. Hence the matching graph is Graph B.

(b) Let P be the amount invested, r be the interest rate per time period, x be the number of time periods and F be the final value. In each case, $P = 100$. Then:

(i) Interest compounds annually so we use the rate and number of time periods given in the question.
Hence $r = 6\% = 0.06$ and $n = 2$, so $F = 100(1 + 0.06)^2 = 100(1.06)^2 = 112.36$.
The final balance is \$112.36.

(ii) Interest compounds twice a year so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 3\% = 0.03$ and $n = 4$, so $F = 100(1 + 0.03)^4 = 100(1.03)^4 = 112.55$.
The final balance is \$112.55.

(iii) Interest compounds 12 times a year so we need to divide the given rate by 12 and multiply the given number of time periods by 12.
Hence $r = 0.5\% = 0.005$ and $n = 24$, so $F = 100(1 + 0.005)^{24} = 100(1.005)^{24} = 112.72$.
The final balance is \$112.72.

(iv) Interest compounds continuously, so $F = 100e^{0.06\times 2} = 100e^{0.12} = 112.75$.
The final balance is \$112.75.

(c) Given an angle a in radians, to convert a to degrees you multiply by 180 and divide by π. Hence the converted angles are:

$$180^\circ \qquad -540^\circ \qquad 450^\circ \qquad -90^\circ \qquad 240^\circ \qquad -495^\circ \qquad 150^\circ \qquad 100^\circ.$$

(d) Given an angle a in degrees, to convert a to radians you divide by 180 and multiply by π. Hence the converted angles are:

$$4\pi \qquad -5\pi \qquad \frac{5\pi}{2} \qquad \frac{-\pi}{2} \qquad \frac{-8\pi}{3} \qquad \frac{-3\pi}{4} \qquad \frac{-5\pi}{6} \qquad \frac{-5\pi}{9}.$$

(e) **(i)** $1000000 = 10^6$, so the answer is 6.

(ii) $1 = 10^{-0}$, so the answer is 0.

(iii) The answer is 1.

(iv) $\dfrac{1}{e^6} = e^{-6}$, so the answer is -6.

(f) The graph of $y_1 = -\sin\dfrac{x}{2}$ is on the left, and the graph of $y_2 = \dfrac{1}{2}\sin 2x$ is on the right. In each case, the graph of $\sin x$ is shaded more lightly.