MATH1040 Solutions to Practice Questions 9

1. **(a)** $y = -3x + 3$, so $f(x) = -3x + 3$, and $f(x+h) = -3(x+h) + 3$. Hence

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{-3(x+h) + 3 - (-3x+3)}{h} \\
&= \lim_{h\to 0} \frac{-3x - 3h + 3 + 3x - 3}{h} \\
&= \lim_{h\to 0} \frac{-3h}{h} \\
&= \lim_{h\to 0} -3 \\
&= -3.
\end{aligned}$$

Hence $y' = -3$.

(b) $y = -2x^2 - 2$, so $f(x) = -2x^2 - 2$, and $f(x+h) = -2(x+h)^2 - 2$. Hence

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{-2(x+h)^2 - 2 - (-2x^2 - 2)}{h} \\
&= \lim_{h\to 0} \frac{-2x^2 - 4xh - 2h^2 - 2 + 2x^2 + 2}{h} \\
&= \lim_{h\to 0} \frac{-4xh - 2h^2}{h} \\
&= \lim_{h\to 0} \frac{h(-4x - 2h)}{h} \\
&= \lim_{h\to 0} -4x - 2h \\
&= -4x.
\end{aligned}$$

Hence $y' = -4x$.

(c) $y = -4x + 6$, so

$$y' = -4.$$

(d) $y = -x^2 + 6x + 5$, so

$$\begin{aligned}
y' &= 2 \times -x^{2-1} + 6 \\
&= -2x + 6.
\end{aligned}$$

(e) $y = 6x^3 - 5x^2 + 6x + 3$, so

$$\begin{aligned}
y' &= 3 \times 6x^{3-1} + 2 \times -5x^{2-1} + 6 \\
&= 18x^2 - 10x + 6.
\end{aligned}$$

(f) $y = 4x^7 - 3x^6 + 2x^3 + 3x$, so

$$\begin{aligned}
y' &= 7 \times 4x^{7-1} + 6 \times -3x^{6-1} + 3 \times 2x^{3-1} + 3 \\
&= 28x^6 - 18x^5 + 6x^2 + 3.
\end{aligned}$$

(g) $y = 2x^2 + 5 + \frac{6}{x} - \frac{1}{x^2}$, so $y = 2x^2 + 5 + 6x^{-1} - x^{-2}$, so

$$\begin{aligned}
y' &= 2 \times 2x^{2-1} - 1 \times 6x^{-1-1} - 2 \times -x^{-2-1} \\
&= 4x - 6x^{-2} + 2x^{-3} \\
&= 4x - \frac{6}{x^2} + \frac{2}{x^3}.
\end{aligned}$$

(h) $y = -x - \frac{1}{x} + \frac{2}{x^2} + \frac{6}{x^4}$, so $y = -x - x^{-1} + 2x^{-2} + 6x^{-4}$, so

$$\begin{aligned} y' &= -1 - 1 \times -x^{-1-1} - 2 \times 2x^{-2-1} - 4 \times 6x^{-4-1} \\ &= -1 + x^{-2} - 4x^{-3} - 24x^{-5} \\ &= -1 + \frac{1}{x^2} - \frac{4}{x^3} - \frac{24}{x^5}. \end{aligned}$$

(i) $y = \sin x + 4\cos x - 4\sqrt{x}$, so $y = \sin x + 4\cos x - 4x^{1/2}$, so

$$\begin{aligned} y' &= \cos x - 4\sin x - \frac{1}{2} \times 4x^{1/2-1} \\ &= \cos x - 4\sin x - 2x^{-1/2} \\ &= \cos x - 4\sin x - \frac{2}{\sqrt{x}}. \end{aligned}$$

(j) $y = 6\ln x - 6e^x$, so

$$\begin{aligned} y' &= 6 \times \frac{1}{x} - 6e^x \\ &= \frac{6}{x} - 6e^x. \end{aligned}$$

(k) $y = 5x^5 - \frac{3}{x} + 2\cos x - 3\sin x + 3\sqrt{x}$, so $y = 5x^5 - 3x^{-1} + 2\cos x - 3\sin x + 3x^{1/2}$, so

$$\begin{aligned} y' &= 5 \times 5x^{5-1} - 1 \times -3x^{-1-1} - 2\sin x - 3\cos x + \frac{1}{2} \times 3x^{1/2-1} \\ &= 25x^4 + 3x^{-2} - 2\sin x - 3\cos x + \frac{3}{2}x^{-1/2} \\ &= 25x^4 + \frac{3}{x^2} - 2\sin x - 3\cos x + \frac{3}{2\sqrt{x}}. \end{aligned}$$

2. (a) $y = -2x + 5$, so $f(x) = -2x + 5$, and $f(x+h) = -2(x+h) + 5$. Hence

$$\begin{aligned} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{-2(x+h) + 5 - (-2x + 5)}{h} \\ &= \lim_{h \to 0} \frac{-2x - 2h + 5 + 2x - 5}{h} \\ &= \lim_{h \to 0} \frac{-2h}{h} \\ &= \lim_{h \to 0} -2 \\ &= -2. \end{aligned}$$

Hence $y' = -2$.

(b) $y = x^2 + 5$, so $f(x) = x^2 + 5$, and $f(x+h) = (x+h)^2 + 5$. Hence

$$\begin{aligned} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{(x+h)^2 + 5 - (x^2 + 5)}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \to 0} \frac{h(2x + h)}{h} \\ &= \lim_{h \to 0} 2x + h \\ &= 2x. \end{aligned}$$

Hence $y' = 2x$.

(c) $y = 5x - 3$, so
$$y' = 5.$$

(d) $y = 4x^2 - x + 4$, so
$$\begin{aligned} y' &= 2 \times 4x^{2-1} - 1 \\ &= 8x - 1. \end{aligned}$$

(e) $y = x^3 - 5x^2 + 3x + 6$, so
$$\begin{aligned} y' &= 3 \times x^{3-1} + 2 \times -5x^{2-1} + 3 \\ &= 3x^2 - 10x + 3. \end{aligned}$$

(f) $y = x^4 - 4x^3 + 6x^2$, so
$$\begin{aligned} y' &= 4 \times x^{4-1} + 3 \times -4x^{3-1} + 2 \times 6x^{2-1} \\ &= 4x^3 - 12x^2 + 12x. \end{aligned}$$

(g) $y = 3x^2 - 1 + \frac{1}{x} - \frac{2}{x^2}$, so $y = 3x^2 - 1 + x^{-1} - 2x^{-2}$, so
$$\begin{aligned} y' &= 2 \times 3x^{2-1} - 1 \times x^{-1-1} - 2 \times -2x^{-2-1} \\ &= 6x - x^{-2} + 4x^{-3} \\ &= 6x - \frac{1}{x^2} + \frac{4}{x^3}. \end{aligned}$$

(h) $y = 5x - \frac{6}{x^2} + \frac{6}{x^4}$, so $y = 5x - 6x^{-2} + 6x^{-4}$, so
$$\begin{aligned} y' &= 5 - 2 \times -6x^{-2-1} - 4 \times 6x^{-4-1} \\ &= 5 + 12x^{-3} - 24x^{-5} \\ &= 5 + \frac{12}{x^3} - \frac{24}{x^5}. \end{aligned}$$

(i) $y = -6\cos x + 4\sin x - 5\sqrt{x}$, so $y = -6\cos x + 4\sin x - 5x^{1/2}$, so
$$\begin{aligned} y' &= 6\sin x + 4\cos x - \frac{1}{2} \times 5x^{1/2-1} \\ &= 6\sin x + 4\cos x - \frac{5}{2}x^{-1/2} \\ &= 6\sin x + 4\cos x - \frac{5}{2\sqrt{x}}. \end{aligned}$$

(j) $y = -6e^x - 5\ln x$, so
$$\begin{aligned} y' &= -6e^x - 5 \times \frac{1}{x} \\ &= -6e^x - \frac{5}{x}. \end{aligned}$$

(k) $y = -4x^2 - \frac{2}{x} + 6e^x + 4\cos x - 6\sin x$, so $y = -4x^2 - 2x^{-1} + 6e^x + 4\cos x - 6\sin x$, so
$$\begin{aligned} y' &= 2 \times -4x^{2-1} - 1 \times -2x^{-1-1} + 6e^x - 4\sin x - 6\cos x \\ &= -8x + 2x^{-2} + 6e^x - 4\sin x - 6\cos x \\ &= -8x + \frac{2}{x^2} + 6e^x - 4\sin x - 6\cos x. \end{aligned}$$

3. **(a)** $y = 4x - 2$, so $f(x) = 4x - 2$, and $f(x+h) = 4(x+h) - 2$. Hence
$$\begin{aligned} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{4(x+h) - 2 - (4x-2)}{h} \\ &= \lim_{h \to 0} \frac{4x + 4h - 2 - 4x + 2}{h} \\ &= \lim_{h \to 0} \frac{4h}{h} \\ &= \lim_{h \to 0} 4 \\ &= 4. \end{aligned}$$

Hence $y' = 4$.

(b) $y = 2x^2 + 1$, so $f(x) = 2x^2 + 1$, and $f(x+h) = 2(x+h)^2 + 1$. Hence

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{2(x+h)^2 + 1 - (2x^2+1)}{h} \\
&= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 + 1 - 2x^2 - 1}{h} \\
&= \lim_{h\to 0} \frac{4xh + 2h^2}{h} \\
&= \lim_{h\to 0} \frac{h(4x+2h)}{h} \\
&= \lim_{h\to 0} 4x + 2h \\
&= 4x.
\end{aligned}$$

Hence $y' = 4x$.

(c) $y = -x - 3$, so

$$y' = -1.$$

(d) $y = -2x^2 + 4x - 6$, so

$$\begin{aligned}
y' &= 2 \times -2x^{2-1} + 4 \\
&= -4x + 4.
\end{aligned}$$

(e) $y = x^3 + 4x^2 - 5x + 2$, so

$$\begin{aligned}
y' &= 3 \times x^{3-1} + 2 \times 4x^{2-1} - 5 \\
&= 3x^2 + 8x - 5.
\end{aligned}$$

(f) $y = 4x^3 - 6x^2 + 3x + 5$, so

$$\begin{aligned}
y' &= 3 \times 4x^{3-1} + 2 \times -6x^{2-1} + 3 \\
&= 12x^2 - 12x + 3.
\end{aligned}$$

(g) $y = 5x^2 + 1 + \frac{2}{x} - \frac{4}{x^2}$, so $y = 5x^2 + 1 + 2x^{-1} - 4x^{-2}$, so

$$\begin{aligned}
y' &= 2 \times 5x^{2-1} - 1 \times 2x^{-1-1} - 2 \times -4x^{-2-1} \\
&= 10x - 2x^{-2} + 8x^{-3} \\
&= 10x - \frac{2}{x^2} + \frac{8}{x^3}.
\end{aligned}$$

(h) $y = 6x^4 + 6x^3 - 1 - \frac{3}{x^3} - \frac{1}{x^4}$, so $y = 6x^4 + 6x^3 - 1 - 3x^{-3} - x^{-4}$, so

$$\begin{aligned}
y' &= 4 \times 6x^{4-1} + 3 \times 6x^{3-1} - 3 \times -3x^{-3-1} - 4 \times -x^{-4-1} \\
&= 24x^3 + 18x^2 + 9x^{-4} + 4x^{-5} \\
&= 24x^3 + 18x^2 + \frac{9}{x^4} + \frac{4}{x^5}.
\end{aligned}$$

(i) $y = -3\sin x - 5\cos x - 4\sqrt{x}$, so $y = -3\sin x - 5\cos x - 4x^{1/2}$, so

$$\begin{aligned}
y' &= -3\cos x + 5\sin x - \frac{1}{2} \times 4x^{1/2-1} \\
&= -3\cos x + 5\sin x - 2x^{-1/2} \\
&= -3\cos x + 5\sin x - \frac{2}{\sqrt{x}}.
\end{aligned}$$

(j) $y = 4\ln x + 5e^x$, so

$$\begin{aligned}
y' &= 4 \times \frac{1}{x} + 5e^x \\
&= \frac{4}{x} + 5e^x.
\end{aligned}$$

(k) $y = 5x^3 - \frac{2}{x} - 6\sqrt{x} + 3e^x - 2\sin x,$ so $y = 5x^3 - 2x^{-1} - 6x^{1/2} + 3e^x - 2\sin x,$ so

$$\begin{aligned} y' &= 3 \times 5x^{3-1} - 1 \times -2x^{-1-1} - \frac{1}{2} \times 6x^{1/2-1} + 3e^x - 2\cos x \\ &= 15x^2 + 2x^{-2} - 3x^{-1/2} + 3e^x - 2\cos x \\ &= 15x^2 + \frac{2}{x^2} - \frac{3}{\sqrt{x}} + 3e^x - 2\cos x. \end{aligned}$$

4. (a) $y = 5x - 2$, so $f(x) = 5x - 2$, and $f(x+h) = 5(x+h) - 2$. Hence

$$\begin{aligned} \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{5(x+h) - 2 - (5x-2)}{h} \\ &= \lim_{h\to 0} \frac{5x + 5h - 2 - 5x + 2}{h} \\ &= \lim_{h\to 0} \frac{5h}{h} \\ &= \lim_{h\to 0} 5 \\ &= 5. \end{aligned}$$

Hence $y' = 5$.

(b) $y = 5x^2 - 1$, so $f(x) = 5x^2 - 1$, and $f(x+h) = 5(x+h)^2 - 1$. Hence

$$\begin{aligned} \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{5(x+h)^2 - 1 - (5x^2 - 1)}{h} \\ &= \lim_{h\to 0} \frac{5x^2 + 10xh + 5h^2 - 1 - 5x^2 + 1}{h} \\ &= \lim_{h\to 0} \frac{10xh + 5h^2}{h} \\ &= \lim_{h\to 0} \frac{h(10x + 5h)}{h} \\ &= \lim_{h\to 0} 10x + 5h \\ &= 10x. \end{aligned}$$

Hence $y' = 10x$.

(c) $y = -4x + 1,$ so

$$y' = -4.$$

(d) $y = 5x^2 - 3x + 5,$ so

$$\begin{aligned} y' &= 2 \times 5x^{2-1} - 3 \\ &= 10x - 3. \end{aligned}$$

(e) $y = -x^3 + 5x^2 - 5x + 4,$ so

$$\begin{aligned} y' &= 3 \times -x^{3-1} + 2 \times 5x^{2-1} - 5 \\ &= -3x^2 + 10x - 5. \end{aligned}$$

(f) $y = x^{12} + 3x^{10} - 2x^7 - 2x^4 + 3x,$ so

$$\begin{aligned} y' &= 12 \times x^{12-1} + 10 \times 3x^{10-1} + 7 \times -2x^{7-1} + 4 \times -2x^{4-1} + 3 \\ &= 12x^{11} + 30x^9 - 14x^6 - 8x^3 + 3. \end{aligned}$$

(g) $y = 2x^2 - 4 - \dfrac{6}{x} + \dfrac{4}{x^2}$, so $y = 2x^2 - 4 - 6x^{-1} + 4x^{-2}$, so

$$\begin{aligned} y' &= 2 \times 2x^{2-1} - 1 \times -6x^{-1-1} - 2 \times 4x^{-2-1} \\ &= 4x + 6x^{-2} - 8x^{-3} \\ &= 4x + \frac{6}{x^2} - \frac{8}{x^3}. \end{aligned}$$

(h) $y = 6 - \dfrac{2}{x} + \dfrac{2}{x^4}$, so $y = 6 - 2x^{-1} + 2x^{-4}$, so

$$\begin{aligned} y' &= -1 \times -2x^{-1-1} - 4 \times 2x^{-4-1} \\ &= 2x^{-2} - 8x^{-5} \\ &= \frac{2}{x^2} - \frac{8}{x^5}. \end{aligned}$$

(i) $y = -4\cos x - 5\sin x - 5\sqrt{x}$, so $y = -4\cos x - 5\sin x - 5x^{1/2}$, so

$$\begin{aligned} y' &= 4\sin x - 5\cos x - \frac{1}{2} \times 5x^{1/2-1} \\ &= 4\sin x - 5\cos x - \frac{5}{2}x^{-1/2} \\ &= 4\sin x - 5\cos x - \frac{5}{2\sqrt{x}}. \end{aligned}$$

(j) $y = -4e^x - 3\ln x$, so

$$\begin{aligned} y' &= -4e^x - 3 \times \frac{1}{x} \\ &= -4e^x - \frac{3}{x}. \end{aligned}$$

(k) $y = -5x^2 - \dfrac{4}{x} + 4\sin x + 4e^x + 3\sqrt{x}$, so $y = -5x^2 - 4x^{-1} + 4\sin x + 4e^x + 3x^{1/2}$, so

$$\begin{aligned} y' &= 2 \times -5x^{2-1} - 1 \times -4x^{-1-1} + 4\cos x + 4e^x + \frac{1}{2} \times 3x^{1/2-1} \\ &= -10x + 4x^{-2} + 4\cos x + 4e^x + \frac{3}{2}x^{-1/2} \\ &= -10x + \frac{4}{x^2} + 4\cos x + 4e^x + \frac{3}{2\sqrt{x}}. \end{aligned}$$

5. (a) $y = -5x - 3$, so $f(x) = -5x - 3$, and $f(x+h) = -5(x+h) - 3$. Hence

$$\begin{aligned} \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{-5(x+h) - 3 - (-5x - 3)}{h} \\ &= \lim_{h\to 0} \frac{-5x - 5h - 3 + 5x + 3}{h} \\ &= \lim_{h\to 0} \frac{-5h}{h} \\ &= \lim_{h\to 0} -5 \\ &= -5. \end{aligned}$$

Hence $y' = -5$.

(b) $y = 4x^2 - 3$, so $f(x) = 4x^2 - 3$, and $f(x+h) = 4(x+h)^2 - 3$. Hence

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0} \frac{4(x+h)^2 - 3 - (4x^2-3)}{h} \\
&= \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 3 - 4x^2 + 3}{h} \\
&= \lim_{h\to 0} \frac{8xh + 4h^2}{h} \\
&= \lim_{h\to 0} \frac{h(8x+4h)}{h} \\
&= \lim_{h\to 0} 8x + 4h \\
&= 8x.
\end{aligned}$$

Hence $y' = 8x$.

(c) $y = 2x - 3$, so

$$y' = 2.$$

(d) $y = 6x^2 + 5x - 4$, so

$$\begin{aligned}
y' &= 2 \times 6x^{2-1} + 5 \\
&= 12x + 5.
\end{aligned}$$

(e) $y = -6x^3 - 4x^2 + 6x - 6$, so

$$\begin{aligned}
y' &= 3 \times -6x^{3-1} + 2 \times -4x^{2-1} + 6 \\
&= -18x^2 - 8x + 6.
\end{aligned}$$

(f) $y = 6x^6 - 5x^3 + 5x$, so

$$\begin{aligned}
y' &= 6 \times 6x^{6-1} + 3 \times -5x^{3-1} + 5 \\
&= 36x^5 - 15x^2 + 5.
\end{aligned}$$

(g) $y = x^2 + 3 + \frac{6}{x} + \frac{4}{x^2}$, so $y = x^2 + 3 + 6x^{-1} + 4x^{-2}$, so

$$\begin{aligned}
y' &= 2 \times x^{2-1} - 1 \times 6x^{-1-1} - 2 \times 4x^{-2-1} \\
&= 2x - 6x^{-2} - 8x^{-3} \\
&= 2x - \frac{6}{x^2} - \frac{8}{x^3}.
\end{aligned}$$

(h) $y = \frac{1}{x} - \frac{5}{x^2} - \frac{4}{x^4}$, so $y = x^{-1} - 5x^{-2} - 4x^{-4}$, so

$$\begin{aligned}
y' &= -1 \times x^{-1-1} - 2 \times -5x^{-2-1} - 4 \times -4x^{-4-1} \\
&= -x^{-2} + 10x^{-3} + 16x^{-5} \\
&= -\frac{1}{x^2} + \frac{10}{x^3} + \frac{16}{x^5}.
\end{aligned}$$

(i) $y = -2\cos x + 5\sin x - 2\sqrt{x}$, so $y = -2\cos x + 5\sin x - 2x^{1/2}$, so

$$\begin{aligned}
y' &= 2\sin x + 5\cos x - \frac{1}{2} \times 2x^{1/2-1} \\
&= 2\sin x + 5\cos x - x^{-1/2} \\
&= 2\sin x + 5\cos x - \frac{1}{\sqrt{x}}.
\end{aligned}$$

(j) $y = -6e^x - \ln x$, so

$$\begin{aligned}
y' &= -6e^x - \frac{1}{x} \\
&= -6e^x - \frac{1}{x}.
\end{aligned}$$

(k) $y = 2x^2 + \dfrac{4}{x} + 5\ln x + 4\sin x - 6\sqrt{x}$, so $y = 2x^2 + 4x^{-1} + 5\ln x + 4\sin x - 6x^{1/2}$, so

$$
\begin{aligned}
y' &= 2 \times 2x^{2-1} - 1 \times 4x^{-1-1} + 5 \times \frac{1}{x} + 4\cos x - \frac{1}{2} \times 6x^{1/2-1} \\
&= 4x - 4x^{-2} + \frac{5}{x} + 4\cos x - 3x^{-1/2} \\
&= 4x - \frac{4}{x^2} + \frac{5}{x} + 4\cos x - \frac{3}{\sqrt{x}}.
\end{aligned}
$$