WEEK 3 PRACTICE SOLUTIONS

1. (a) $\dfrac{x^0x^{-3}}{x^2x^0} = \dfrac{x^{0-3}}{x^{2+0}} = \dfrac{x^{-3}}{x^2} = x^{-3-2} = x^{-5}$.

(b)
$$\begin{aligned} x^1x^1y^{-1} \div (x^{-1}y^0) \times y^2 &= x^1x^1y^{-1} \times x^1y^0 \times y^2 \\ &= x^{1+1+1}y^{-1+0+2} \\ &= x^3y^1. \end{aligned}$$

(c)
$$\begin{aligned} 3x+2 &< -3x-1 \\ 3x+2-2 &< -3x-1-2 \\ 3x &< -3x-3 \\ 3x+3x &< -3x-3+3x \\ 6x &< -3 \\ 6x \div 6 &< -3 \div 6 \\ x &< -\frac{1}{2}. \end{aligned}$$

In inequality form, the answer is $(-\infty, -\frac{1}{2})$.

(d) $\displaystyle\sum_{i=-2}^{1} 3ix = -6x - 3x + 0x + 3x = -6x.$

(e)
$$\begin{aligned} \sum_{i=2}^{5}(-1)^i i &= (-1)^2 \times 2 + (-1)^3 \times 3 + (-1)^4 \times 4 + (-1)^5 \times 5 \\ &= 2 - 3 + 4 - 5 = -2. \end{aligned}$$

2. (a) (1) $A = \{1,2,3\}$.

(2) $A \cup B = \{0,1,2,3,8,10\}$.

(3) $B \cap C = \{0,8\}$.

(4) $A \setminus C = \{1,3\}$.

(5) $A \setminus (B \cup C) = \{3\}$.

(6) $(A \cap B) \cap C = \{\}$.

(b) (1) $A \cap B = \{-1,0,4\} \cap \{-2,0,1\} = \{0\}$

(2) $B \cup \emptyset = \{-2,0,1\} \cup \{\} = \{-2,0,1\}$

(3)
$$\begin{aligned} C \cap (B \setminus A) &= C \cap (\{-2,0,1\} \setminus \{-1,0,4\}) \\ &= \{-1,0,5\} \cap \{-2,1\} = \{\} \end{aligned}$$

(4)
$$\begin{aligned} (B \cup A) \cup C &= (\{-2,0,1\} \cup \{-1,0,4\}) \cup C \\ &= \{-2,-1,0,1,4\} \cup \{-1,0,5\} = \{-2,-1,0,1,4,5\} \end{aligned}$$

(5)
$$\begin{aligned} (C \cup \emptyset) \cap B &= (\{-1,0,5\} \cup \{\}) \cap B \\ &= \{-1,0,5\} \cap \{-2,0,1\} = \{0\} \end{aligned}$$

(6)
$$\begin{aligned} (B \cap C) \cap (B \cup C) &= (\{-2,0,1\} \cap \{-1,0,5\}) \cap (\{-2,0,1\} \cup \{-1,0,5\}) \\ &= \{0\} \cap \{-2,-1,0,1,5\} = \{0\}. \end{aligned}$$

(7)
$$\begin{aligned} (A \setminus \emptyset) \setminus (B \setminus \emptyset) &= (\{-1,0,4\} \setminus \{\}) \setminus (\{-2,0,1\} \setminus \{\}) \\ &= \{-1,0,4\} \setminus \{-2,0,1\} = \{-1,4\}. \end{aligned}$$

(c) (1) $p = 5/10 = 1/2$.

(2) $p = 6/10 = 3/5$.

(3) $p = 3/10$.

(4) $p = 8/10 = 4/5$.

(5) $p = 3/6 = 1/2$.

(6) Prob(r_1 is even) equals 5/10, and Prob(r_2 is even) equals 4/8. Now r_1 and r_2 are chosen independently, so Prob(both even) = Prob(r_1 is even) $\times$ Prob(r_2 is even).

Hence $p = \dfrac{5}{10} \times \dfrac{4}{8} = \dfrac{20}{80} = \dfrac{1}{4}$.

(7) By the principle of inclusion/exclusion,

Prob(r_1 even or r_2 even) = Prob(r_1 even)+Prob(r_2 even)$-$ Prob(r_1 **and** r_2 are even).

Hence $p = \dfrac{5}{10} + \dfrac{4}{8} - \dfrac{20}{80} = \dfrac{40+40-20}{80} = \dfrac{60}{80} = \dfrac{3}{4}$.

(8) Now r_1 and r_2 are chosen independently, so

Prob(r_1 is even **given that** r_2 is even) =Prob(r_1 is even).

Hence $p = 5/10 = 1/2$.