WEEK 5 SOLUTIONS

1. (a) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -x & - & 5y & = & 29 & (1) \\ 5x & - & y & = & -15 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (1) gives

$$x = -5y - 29 \quad (3)$$

Substituting equation (3) into equation (2) gives

$$5(-5y - 29) - y = -15. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$-25y - 145 - y = -15, \quad (5)$$

and simplifying equation (5) gives

$$-26y - 145 = -15. \quad (6)$$

Then solving equation (6) gives $-26y = 130$ so $y = \dfrac{130}{-26} = -5.$

Next we substitute the value for y into equation (3) to obtain the value for x, giving

$$x = -5 \times -5 - 29 \quad \text{so } x = -4.$$

Hence the simultaneous solution to equations (1) and (2) is $x = -4$ and $y = -5$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -x - 5y = -1 \times -4 - 5 \times -5 = 4 + 25 = 29$, as required.

(2) $\rightarrow 5x - y = 5 \times -4 - 1 \times -5 = -20 + 5 = -15$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 3x & - & y & = & -5 & (1) \\ -3x & + & y & = & -3 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$3x - 3x - y + y = -5 - 3. \quad (3)$$

Simplifying equation (3) gives

$$0 + 0 = -8. \quad (4)$$

Then solving equation (4) gives $0 = -8$.

This statement is **never** true, so there is no solution to the given equations.

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -5x & - & y & = & 7 & (1) \\ -5x & - & y & = & -7 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by -1, giving

$$\begin{array}{rcrcrl} 5x & + & y & = & -7 & (3) \\ -5x & - & y & = & -7 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$5x - 5x + y - y = -7 - 7. \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = -14. \quad (6)$$

Then solving equation (6) gives $0 = -14$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -7x & + & 6y & = & -7 & (1) \\ 7x & - & 6y & = & 7 & (2) \end{array}$$

Now we add both sides of equations (1) and (2), giving

$$-7x + 7x + 6y - 6y = -7 + 7. \quad (3)$$

Simplifying equation (3) gives

$$0 + 0 = 0. \quad (4)$$

Then solving equation (4) gives $\quad 0 = 0$.

This statement is always true, so there is an infinite number of solutions to the given equations.

2. (a) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -2x & + & 2y & = & 4 & (1) \\ -5x & - & 5y & = & 30 & (2) \end{array}$$

Next we take out any common factors.

$$\begin{array}{rcrcrl} -x & + & y & = & 2 & (3) \\ -x & - & y & = & 6 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$-x - x + y - y = 2 + 6. \quad (5)$$

Simplifying equation (5) gives

$$-2x + 0 = 8. \quad (6)$$

Then solving equation (6) gives $\quad -2x = 8 \quad$ so $\quad x = \frac{8}{-2} \quad = -4.$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$-1 \times -4 + y = 2 \quad \text{so} \quad 1y = -2 \ .$$

Hence the simultaneous solution to equations (1) and (2) is $x = -4$ and $y = -2$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -2x + 2y = -2 \times -4 + 2 \times -2 = 8 - 4 = 4$, as required.

(2) $\rightarrow -5x - 5y = -5 \times -4 - 5 \times -2 = 20 + 10 = 30$, as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

$$\begin{array}{rcrcrl} -2x & - & y & = & 5 & (1) \\ x & + & 5y & = & -5 & (2) \end{array}$$

We solve these using substitution. Rearranging equation (1) gives

$$y = -2x - 5 \quad (3)$$

Substituting equation (3) into equation (2) gives

$$5(-2x - 5) + x = -5. \quad (4)$$

Then expanding the brackets in equation (4) gives

$$-10x - 25 + x = -5, \quad (5)$$

and simplifying equation (5) gives

$$-9x - 25 = -5. \quad (6)$$

Then solving equation (6) gives $\quad -9x = 20 \quad$ so $\quad x = \frac{20}{-9} \quad = -\frac{20}{9}.$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = -2 \times \frac{-20}{9} - 5 \quad \text{so } y = \frac{-5}{9} = -\frac{5}{9}.$$

Hence the simultaneous solution to equations (1) and (2) is $x = -\frac{20}{9}$ and $y = -\frac{5}{9}$.

(As always, check your answers by substituting into equations (1) and (2).
(1) $\rightarrow -2x - y = -2 \times -\frac{20}{9} - 1 \times -\frac{5}{9} = \frac{40}{9} + \frac{5}{9} = 5$, as required.

(2) $\rightarrow x + 5y = 1 \times -\frac{20}{9} + 5 \times -\frac{5}{9} = \frac{-20}{9} - \frac{25}{9} = -5$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 4x & - & 7y & = & -3 & (1) \\ 4x & - & 7y & = & -6 & (2) \end{array}$$

We solve these using elimination. Multiply equation (1) by -1, giving

$$\begin{array}{rcrcrl} -4x & + & 7y & = & 3 & (3) \\ 4x & - & 7y & = & -6 & (4) \end{array}$$

Now we add both sides of equations (3) and (4), giving

$$-4x + 4x + 7y - 7y = 3 - 6. \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = -3. \quad (6)$$

Then solving equation (6) gives $0 = -3$.

This statement is **never** true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$\begin{array}{rcrcrl} 6x & + & 4y & = & -2 & (1) \\ 3x & + & 2y & = & -1 & (2) \end{array}$$

Next we take out any common factors.

$$\begin{array}{rcrcrl} 3x & + & 2y & = & -1 & (3) \\ 3x & + & 2y & = & -1 & (4) \end{array}$$

We solve these using elimination. Multiply equation (3) by -1, giving

$$\begin{array}{rcrcrl} -3x & - & 2y & = & 1 & (5) \\ 3x & + & 2y & = & -1 & (6) \end{array}$$

Now we add both sides of equations (5) and (6), giving

$$-3x + 3x - 2y + 2y = 1 - 1. \quad (7)$$

Simplifying equation (7) gives

$$0 + 0 = 0. \quad (8)$$

Then solving equation (8) gives $0 = 0$.

This statement is **always** true, so there is an infinite number of solutions to the given equations.

3. (a) 2 points on the track are $(x_1, y_1) = (-3, 10)$ and $(x_2, y_2) = (1, 2)$. So $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-10}{1+3} = \frac{-8}{4} = -2$. Hence $y = -2x + c$. Now $(1,2)$ on the line, so
$2 = -2 \times 1 + c \Rightarrow \quad c = 4 \Rightarrow \quad y = -2x + 4$.

(b) Points $(x_1, y_1) = (-1, 6)$ and $(x_2, y_2) = (-3, 2)$. So $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{-3+1} = \frac{-4}{-2} = 2$. Hence $y = 2x + c$. Now $(-3, 2)$ on the line, so $2 = 2 \times -3 + c$
$\Rightarrow \quad 2 = -6 + c \Rightarrow \quad c = 8 \Rightarrow \quad y = 2x + 8$.

(c) If Sassie's ride is parallel to Dagwood's, then the slope of Sassie's ride must also have slope $m = 2$. So $y = 2x + c$. Now Sassie starts at $(-2, 0)$, so $0 = 2 \times -2 + c \Rightarrow \quad c = 4$, so $y = 2x + 4$. To find the crossing point, 2 equations: $y = 2x + 4$ (Sassie) and $y = -2x + 4$ (train). Hence $2x + 4 = -2x + 4 \Rightarrow \quad 4x = 0 \Rightarrow \quad x = 0$. Therefore $y = 2 \times 0 + 4$, so $y = 4$. Sassie crosses the railway line at $(0, 4)$.

4. (a) Let $x = \#$ whiskeys and $y = \#$ wieners.
So $y = x + 3$ (1) and $6x + 2y = 54$ (2)
Substitute (1) into (2), $6x + 2(x+3) = 54 \Rightarrow \quad 6x + 2x + 6 = 54 \Rightarrow \quad 8x = 48 \Rightarrow \quad x = 6$ and $y = 9$. She buys 6 whiskies and 9 wieners. Check!

(b) (i) $x + y + z = 24$ (1) $\quad y = \frac{1}{3}(x+z)$ (2) $\quad x = 2z$ (3)

(ii) Now these are 3 equations and 3 unknowns, but we solve in the usual way. Use (3) to substitute into (1) and (2):
$(1) \Rightarrow 2z + y + z = 24 \Rightarrow \quad y + 3z = 24$ (4)
$(2) \Rightarrow y = \frac{1}{3}(z + 2z) = \frac{1}{3}(3z) = z \Rightarrow \quad y = z$ (5)
Now substitute (5) into (4) $\Rightarrow z + 3z = 24 \Rightarrow \quad 4z = 24 \Rightarrow \quad z = 6$. Therefore from (5) $y = 6$. Substitute $z = 6$ and $y = 6$ into (1) $\Rightarrow x + 6 + 6 = 24 \Rightarrow \quad x = 12$.

She spends 12 hours studying, 6 hours sleeping and 6 hours writing poetry.

Finally, you should check these answers by substituting into the three original equations. When you do so, you'll see that they are all true.