WEEK 6 SOLUTIONS

1 (a) $f(1) = 1^2 + 1 = 1 + 1 = 2$.

(b) $f(4) = (-4)^2 + 4 = 16 + 4 = 20$.

(c) $f(1) = -(1^2) + 1 = -1 + 1 = 0$.

(d) Any value of x can be substituted into f, so the domain is $x \in (-\infty, \infty)$.

(e) Square root only works on positive numbers and the denominator of a fraction cannot be 0, so the domain is $x \in (0, \infty)$.

(f) Any value of x can be substituted into f, so the domain is $x \in (-\infty, \infty)$.

(g) The numerator is negative and the denominator is positive, so it is not possible to get a positive value for y. Also, it is not possible to have $y = 0$. Hence the range is $y \in (-\infty, 0)$.

(h) $x^2 \geq 0$ so $-2x^2 \leq 0$, so $-2x^2 + 8 \leq 8$, so the range is $(-\infty, 8]$.

(i) Square root is always positive (or 0), so the range is $y \in [11, \infty)$.

(j) To solve each of these, remember that if $a \times b = 0$ then either $a = 0$ or $b = 0$. Then:

(i) $4x(-2x-1) = 0$. Hence (1) $4x = 0$ or (2) $(-2x-1) = 0$. Solve these:

(1) $x = 0$.

(2) $-2x = 1$, so $x = \frac{1}{-2}$, so $x = -\frac{1}{2}$.

Hence the solutions are $x = 0$ and $x = -\frac{1}{2}$.

(ii) $(-2x-3)(3x+2) = 0$. Hence (1) $-2x - 3 = 0$ or (2) $3x + 2 = 0$. Solve these:

(1) $-2x = 3$, so $x = \frac{3}{-2}$, so $x = -\frac{3}{2}$.

(2) $3x = -2$, so $x = \frac{-2}{3}$, so $x = -\frac{2}{3}$.

Hence the solutions are $x = -\frac{3}{2}$ and $x = -\frac{2}{3}$.

(iii) $3(-x+3)(2x-2) = 0$. Hence (1) $-x + 3 = 0$ or (2) $2x - 2 = 0$. Solve these:

(1) $-x = -3$, so $x = 3$.

(2) $2x = 2$, so $x = \frac{2}{2}$, so $x = 1$.

Hence the solutions are $x = 3$ and $x = 1$.

(iv) $(x+2)^2 = 0$. Hence $x + 2 = 0$, so $x = -2$.

(k) $2x^2 - 3x + 2 = 0$, so we use $a = 2$, $b = -3$ and $c = 2$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{3 \pm \sqrt{(-3)^2 - (4 \times 2 \times 2)}}{2 \times 2} \\ &= \frac{3 \pm \sqrt{9 - 16}}{4} \\ &= \frac{3 \pm \sqrt{-7}}{4}. \end{aligned}$$

Hence there is no solution.

(l) $-2x^2 - 6x - 4 = 0$, so we use $a = -2$, $b = -6$ and $c = -4$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{6 \pm \sqrt{(-6)^2 - (4 \times -2 \times -4)}}{2 \times -2} \\ &= \frac{6 \pm \sqrt{36 - 32}}{-4} \\ &= \frac{6 \pm \sqrt{4}}{-4} \\ &= \frac{6 \pm 2}{-4} \\ &= \frac{6+2}{-4} \text{ or } \frac{6-2}{-4} \\ &= \frac{8}{-4} \text{ or } \frac{4}{-4} \\ &= -2 \text{ or } -1. \end{aligned}$$

2. (a) $f(-3) = (-(-3))^2 + (-3) = 9 - 3 = 6$.

(b) $f(4) = -(4^2) + 4 = -16 + 4 = -12$.

(c) $f(-1) = -(-1)^2 - (-1) = -1 + 1 = 0$.

(d) Any value of x can be substituted into f, so the domain is $x \in (-\infty, \infty)$.

(e) Square root only works on positive numbers and the denominator of a fraction cannot be 0, so the domain is $x \in (14, \infty)$.

(f) It is not possible to have 0 as the denominator of a fraction. This happens when $x - 5 = 0$, so $x = 5$. Hence the domain is **everything else**, so $x \in (-\infty, 5) \cup (5, \infty)$.

(g) Square root is always positive (or 0), so the range is $y \in [-12, \infty)$.

(h) $\sqrt{x}$ is always positive (or 0), so $f(x) = -12\sqrt{x}$ is always negative (or 0), so the range is $y \in (-\infty, 0]$.

(i) $x^2 \geq 0$ so $2x^2 \geq 0$, so $2x^2 + 2 \leq 2$, so the range is $[2, \infty)$.

(j) To solve each of these, remember that if $a \times b = 0$ then either $a = 0$ or $b = 0$. Then:

(i) $4x(x-2) = 0$. Hence (1) $4x = 0$ or (2) $(x - 2) = 0$. Solve these:

(1) $x = 0$.
(2) $x = 2$.

Hence the solutions are $x = 0$ and $x = \frac{2}{1}$.

(ii) $(-3x - 3)(2x + 2) = 0$. Hence (1) $-3x - 3 = 0$ or (2) $2x + 2 = 0$. Solve these:

(1) $-3x = 3$, so $x = \frac{3}{-3}$, so $x = -1$.

(2) $2x = -2$, so $x = \frac{-2}{2}$, so $x = -1$.

Hence the solutions are $x = -1$ and $x = -1$.

(iii) $4(4x - 3)(-x - 2) = 0$. Hence (1) $4x - 3 = 0$ or (2) $-x - 2 = 0$. Solve these:

(1) $4x = 3$, so $x = \frac{3}{4}$.

(2) $-x = 2$, so $x = -2$.

Hence the solutions are $x = \frac{3}{4}$ and $x = -2$.

(iv) $(-2x + 1)^2 = 0$. Hence $-2x + 1 = 0$, so $-2x = -1$, so $x = \frac{-1}{-2}$, so $x = \frac{1}{2}$.

(k) $x^2 + x + 3 = 0$, so we use $a = 1$, $b = 1$ and $c = 3$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1^2 - (4 \times 1 \times 3)}}{2 \times 1} \\ &= \frac{-1 \pm \sqrt{1 - 12}}{2} \\ &= \frac{-1 \pm \sqrt{-11}}{2}. \end{aligned}$$

Hence there is no solution.

(l) $4x^2 + 5x + 1 = 0$, so we use $a = 4$, $b = 5$ and $c = 1$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - (4 \times 4 \times 1)}}{2 \times 4} \\ &= \frac{-5 \pm \sqrt{25 - 16}}{8} \\ &= \frac{-5 \pm \sqrt{9}}{8} \\ &= \frac{-5 \pm 3}{8} \\ &= \frac{-5 + 3}{8} \text{ or } \frac{-5 - 3}{8} \\ &= \frac{-2}{8} \text{ or } \frac{-8}{8} \\ &= -\frac{1}{4} \text{ or } -1. \end{aligned}$$