MATH1040 Summer Assignment 4

All questions should be submitted by 2pm on Thursday 20 December. Assignments can be submitted at your tutorial or to the MATH1040 assignment box (3rd floor, Priestley Building). **Make sure that your name, student number and assignment number are on each sheet of your answers.** Write your answers on a separate sheet of paper. You do not need a cover sheet nor do you need to include this question sheet. Solutions will be distributed in class later.

1. Answer each of the following questions, showing all working:

   (1) Given two sets:
   $C=\{5,7,0,-2\}$ and $B=\{3,5,7,2,0,-2,8,6\}$,
   find $C \cap B$.
   Illustrate your answer with Venn diagram.

   (2) Given two sets $A=\{3,5,2,0,9,-2,4,1,6\}$ and $G=\{3,7,9,4,6\}$,
   find:
   - i. $A \cap G$
   - ii. $A \cup G$
   - iii. $A \backslash G$
   - iv. $G \backslash A$

   (3) For the following questions let $H=\{7,2,9,4\}$, $B=\{-3,-1,1\}$, $A=\{x \mid x \in \mathbb{N}, 7 \leq x \leq 9\}$
   - i. Write down the elements of set H .
   - ii. Write down the elements of the set $B \cup H$.
   - iii. Write down the elements of the set $H \cap A$.
   - iv. Write down the elements of the set $H \backslash A$.
   - v. Write down the elements of the set $B \backslash (H \cup A)$.
   - vi. Write down the elements of the set $(B \backslash A) \backslash H$.
   - vii. Write down the elements of the set $B \cap (H \backslash A)$.
   - viii. Write down the elements of the set $A \cup \emptyset$.
   - ix. Write down the elements of the set $(A \backslash B) \cap (A \cap H)$.

   (4) For the following questions let t_1 and t_2 be random natural numbers chosen independently, where t_1 is between 1 and 7 (inclusive), and t_2 is between 6 and 8 (inclusive). In each case, find the probability p that:
   - i. t_1 is odd?
   - ii. $t_1 = 4$?
   - iii. $t_1 > 3$?
   - iv. t_1 is odd and $t_1 > 3$?
   - v. t_1 is odd or $t_1 > 3$?
   - vi. t_1 is odd given that $t_1 > 3$?
   - vii. Both t_1 and t_2 are odd ?
   - viii. At least one of t_1 and t_2 is odd ?
   - ix. t_1 is odd given that t_2 is even ?

2. Answer each of the following questions, showing all working:

   (1) Find the distance between the points $(-8,-3)$ and $(9,-1)$.

   (2) Find the gradient and y-intercept of the line $0 = 6 - 3y - 2x$.

   (3) Find the equation of the straight line passing through the points $(-8,8)$ and $(6,-9)$.

   (4) Find the equation of the line parallel to $0 = -8y - 40 - 32x$ and passing through the point $(5,-29)$.

   (5) Find the equation of the line perpendicular to $4 - 4y = 8x$ and passing through the point $(-6,-11)$.

   (6) Does the line $-16x + 16 + 4y = 0$ pass through the point $(-2,-12)$?

   (7) Find the equation of the line perpendicular to $0 = 9x + 6$ and passing through the point $(6,5)$.

3. Cassanova loves Cassandra. Alas, her father Baron Bruce de Burgundy does not approve, and shuts her in a tower. Cassanova looks sadly up at her window. After a while, his rampant hormones become irresistible, so he buys a ladder.

   (a) Cassanova could only afford a ladder $2\sqrt{10}$ metres long. He leans it against the wall, with the ladder's base 2 metres out from the wall. If her window is exactly 7 metres above the ground, can he reach her? (Hint: first work out how high his ladder reaches up the wall. Assume he is scared of heights, and can only get in if his ladder exactly reaches the height of the window.)

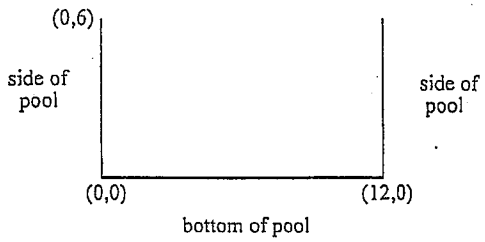
   (b) Cassanova is not fussy: Cassandra has a younger sister, Cecelia. When the bottom of the ladder is placed at the point $(0, 0)$, and the ladder has a gradient of 2 (note: 2, not -2), the top of the ladder just reaches Cecilia's window. Find the equation which describes the ladder.

   (c) If the distance from the origin to the bottom of the building is $2\sqrt{2}$ metres, find the height of Cecilia's window.

   (d) When Cassanova climbs in the window, quietly singing Italian love songs, he receives a surprise: it is not Cecelia, but is instead Baron Bruce, dressed in Cecelia's clothes. Cassanova is still not fussy and carries the Baron down the ladder. Alas, their combined weight is too much, and when they are **halfway** down, the ladder breaks and they fall vertically. Find the equation of the line along which they fall.

4. Stolichnaya Springborg supervises the Slovakian synchronised swimming squad for the Beijing Olympics. He develops a mathematical model of the swimming pool, allowing him to precisely define the paths his synchronised swimmers need to follow.

   The pool is 6 metres deep and 12 metres wide. He assigns coordinates to the pool's cross-section as shown in the following diagram; thus, one unit equals one metre. In this question, assume all movement follows a straight line. Also, all co-ordinates are from a cross-section of the pool, so movement is in two dimensions.



   (a) The star swimmer Stephanie Smirnov is at a point one metre from the left edge of the pool and 2 metres from the bottom. To satisfy the swim scheme, she has to swim to a point on the surface, 3 metres from the left edge. Draw a diagram, showing Stephanie's start point and finish point. Find the equation of the straight line which she must swim.

   (b) The South Saskatchewan swim squad owns the pool, and they are afraid that the Slovakian swimmers will win the gold medal. They decide to cheat, by installing a torpedo tube in the bottom right corner of the pool. A torpedo is aimed directly at the point where Stephanie will reach the surface. Find the equation of the line which the torpedo will follow.

   (c) Stephanie swims at $\sqrt{5}$ metres per second. The torpedo travels at $3\sqrt{13}$ metres per second. To create maximum impact with the judges (not to mention with Stephanie!), the torpedo should reach the point on the surface at exactly the same time Stephanie does. If she starts swimming at $t = 0$ seconds, at what time should the torpedo be fired? (Hint: first calculate how far she has to swim, then how far the torpedo has to travel. Also, $117 = 9 \times 13$.)

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