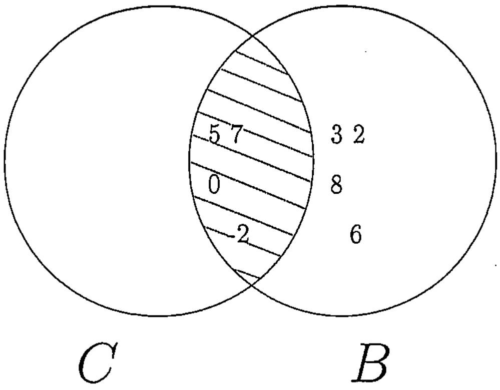
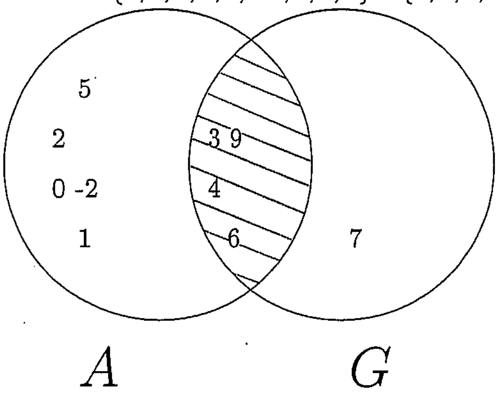
MATH1040 Summer Assignment 4 Solutions

1. (1) $C \cap B = \{5, 7, 0, -2\} \cap \{3, 5, 7, 2, 0, -2, 8, 6\} = \{5, 7, 0, -2\}$
On Venn diagram:

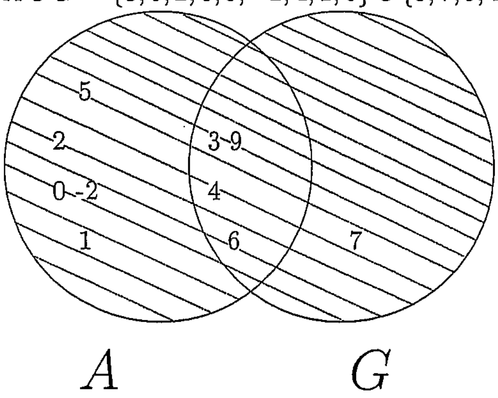


(2) **i.** $A \cap G = \{3, 5, 2, 0, 9, -2, 4, 1, 6\} \cap \{3, 7, 9, 4, 6\} = \{3, 9, 4, 6\}$



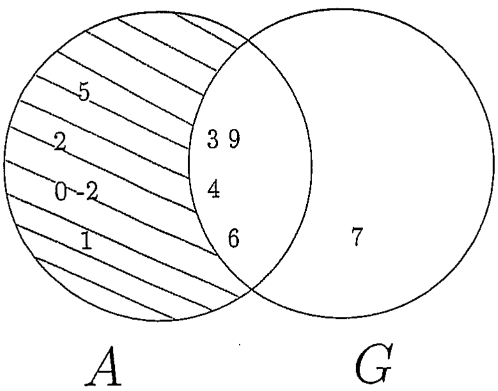
(optional)

ii. $A \cup G = \{3, 5, 2, 0, 9, -2, 4, 1, 6\} \cup \{3, 7, 9, 4, 6\} = \{3, 5, 7, 2, 0, 9, -2, 4, 1, 6\}$



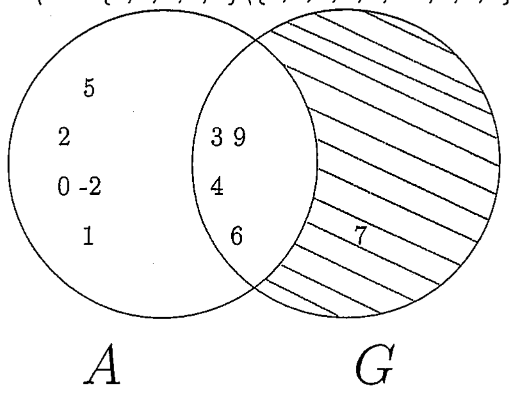
(optional)

iii. $A \backslash G = \{3, 5, 2, 0, 9, -2, 4, 1, 6\} \backslash \{3, 7, 9, 4, 6\} = \{5, 2, 0, -2, 1\}$



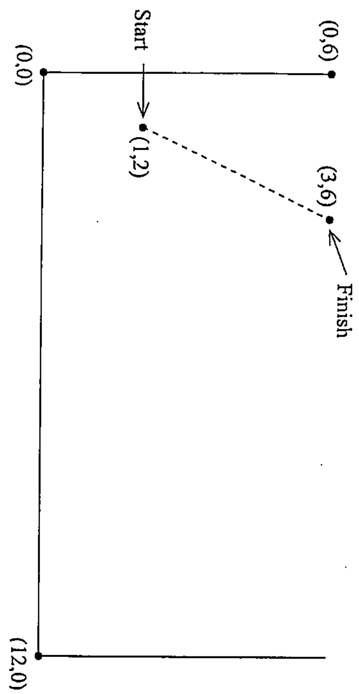
(optional)

iv. $G \backslash A = \{3, 7, 9, 4, 6\} \backslash \{3, 5, 2, 0, 9, -2, 4, 1, 6\} = \{7\}$



(optional)

Q 4a



(3) i. $H = \{7, 2, 9, 4\}$

ii. $B \cup H = \{-3, -1, 1\} \cup \{7, 2, 9, 4\} = \{-3, -1, 7, 2, 9, 4, 1\}$

iii. $H \cap A = \{7, 2, 9, 4\} \cap \{7, 9, 8\} = \{7, 9\}$

iv. $H \backslash A = \{7, 2, 9, 4\} \backslash \{7, 9, 8\} = \{2, 4\}$

v.

$$\begin{aligned} B \backslash (H \cup A) &= \{-3, -1, 1\} \backslash (\{7, 2, 9, 4\} \cup \{7, 9, 8\}) \\ &= \{-3, -1, 1\} \backslash \{7, 2, 9, 4, 8\} \\ &= \{-3, -1, 1\} \end{aligned}$$

vi.

$$\begin{aligned} (B \backslash A) \backslash H &= (\{-3, -1, 1\} \backslash \{7, 9, 8\}) \backslash \{7, 2, 9, 4\} \\ &= \{-3, -1, 1\} \backslash \{7, 2, 9, 4\} \\ &= \{-3, -1, 1\} \end{aligned}$$

vii.

$$\begin{aligned} B \cap (H \backslash A) &= \{-3, -1, 1\} \cap (\{7, 2, 9, 4\} \backslash \{7, 9, 8\}) \\ &= \{-3, -1, 1\} \cap \{2, 4\} \\ &= \emptyset \end{aligned}$$

viii. $A \cup \emptyset = \{7, 9, 8\} \cup \emptyset = \{7, 9, 8\}$

ix.

$$\begin{aligned} (A \backslash B) \cap (A \cap H) &= (\{7, 9, 8\} \backslash \{-3, -1, 1\}) \cap (\{7, 9, 8\} \cap \{7, 2, 9, 4\}) \\ &= \{7, 9, 8\} \cap \{7, 9\} \\ &= \{7, 9\} \end{aligned}$$

(4) i. $Prob\,(t_1 \text{ is odd}) = \frac{4}{7}$

ii. $Prob\,(t_1 = 4) = \frac{1}{7}$

iii. $Prob\,(t_1 > 3) = \frac{4}{7}$

iv. $Prob\,(t_1 \text{ is odd and } t_1 > 3) = \frac{2}{7}$

v. $Prob\,(t_1 \text{ is odd or } t_1 > 3) = \dfrac{6}{7}$

vi. $Prob\,(t_1 \text{ is odd given that } t_1 > 3) = \dfrac{2}{4} = \dfrac{1}{2}$

vii. $Prob\,(t_1 \text{ is odd}) = \dfrac{4}{7}$, and $Prob\,(t_2 \text{ is odd}) = \dfrac{1}{3}$.

Now t_1 and t_2 are chosen independently,

so $Prob\,(\text{both } t_1 \text{ and } t_2 \text{ are odd}) = Prob\,(t_1 \text{ is odd}) \times Prob\,(t_2 \text{ is odd})$.

Hence $Prob\,(\text{both } t_1 \text{ and } t_2 \text{ are odd}) = \dfrac{4}{7} \times \dfrac{1}{3} = \dfrac{4}{21}$

viii. By the principle of inclusion\exclusion,

$Prob\,(t_1 \text{ is odd } \mathbf{or} \ t_2 \text{ is odd}) = Prob\,(t_1 \text{ is odd}) + Prob\,(t_2 \text{ is odd}) - Prob\,(\text{both } t_1 \text{ and } t_2 \text{ are odd})$.

Hence $Prob\,(t_1 \text{ is odd } \mathbf{or} \ t_2 \text{ is odd}) = \dfrac{4}{7} + \dfrac{1}{3} - \dfrac{4}{21} = \dfrac{5}{7}$

ix. Now t_1 and t_2 are chosen independently, so

$Prob\,(t_1 \text{ is odd } \mathbf{given\ that} \ t_2 \text{ is even}) = Prob\,(t_1 \text{ is odd})$.

Hence $Prob\,(t_1 \text{ is odd } \mathbf{given\ that} \ t_2 \text{ is even}) = \dfrac{4}{7}$

2. (1) Let $(x_1, y_1) = (-8, -3)$ and $(x_2, y_2) = (9, -1)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so

$d = \sqrt{(-8-9)^2 + (-3-(-1))^2} = \sqrt{(-17)^2 + (-2)^2} = \sqrt{289+4} = \sqrt{293}$.

Hence $d = \sqrt{293}$

(2) Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 0 &= 6 - 3y - 2x, \quad \text{so} \\ 3y &= -2x + 6 \\ y &= -\frac{2}{3}x + 2 \end{aligned}$$

Hence the gradient is $m = -\dfrac{2}{3}$ and the y-intercept is $c = 2$.

(3) Let $(x_1, y_1) = (-8, 8)$ and $(x_2, y_2) = (6, -9)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-9-8}{6-(-8)} = \dfrac{-17}{14}$. Hence $m = -\dfrac{17}{14}$.

Thus the equation of the line is $y = -\dfrac{17}{14}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-8, 8)$ into this equation to get the value for c.

Hence $8 = -\dfrac{17}{14} \times (-8) + c$, so $8 = \dfrac{68}{7} + c$. Hence $c = 8 - \dfrac{68}{7} = -\dfrac{12}{7}$.

Hence the equation of the line is $y = -\dfrac{17}{14}x - \dfrac{12}{7}$.

(4) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 0 &= -8y - 40 - 32x, \text{ so} \\ 8y &= -32x - 40 \\ y &= -4x - 5 \end{aligned}$$

Hence, the gradient of the original line is $m = -4$.

The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = -4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (5, -29)$ into this equation to get the value for c.

$-29 = -4 \times 5 + c$, so $-29 = -20 + c$. Hence $c = -29 - (-20) = -9$.

Hence the equation of the line is $y = -4x - 9$.

(5) To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 4 - 4y &= 8x, \text{ so} \\ -4y &= 8x - 4 \\ y &= -2x + 1 \end{aligned}$$

Hence the gradient of the original line is $m_0 = -2$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = \frac{1}{2}$.
Thus the equation of the line is $y = \frac{1}{2}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-6, -11)$ into this equation to get the value of c:
$-11 = \frac{1}{2} \times (-6) + c$, so $-11 = -3 + c$. Hence $c = -11 - (-3) = -8$.
Hence the equation of the line is $y = \frac{1}{2}x - 8$.

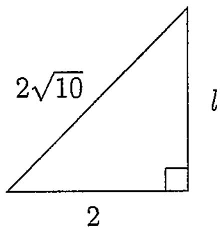
(6) To determine whether the given line passes through the point $(x_1, y_1) = (-2, -12)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} -16x + 16 + 4y &= 0, \text{ so} \\ -16 \times (-2) + 16 + 4 \times (-12) &= 0 \\ 32 + 16 - 48 &= 0 \\ 0 &= 0 \end{aligned}$$

The last statement is **true**, so our line **does** pass through the point $(-2, -12)$.

(7) The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(6, 5)$ lies on the new line, so the equation of the new line is $y = 5$.

3. (a) $(2\sqrt{10})^2 = 2^2 + l^2 \Rightarrow 40 = 4 + l^2 \Rightarrow l^2 = 36 \Rightarrow l = 6$. Window is 7m high, so he **cannot** reach.



(b) $y = 2x + c$, $(0, 0)$ is on the line $\Rightarrow$ $0 = 2 \times 0 + c \Rightarrow$ $c = 0 \Rightarrow$ $y = 2x$

(c) We know the equation of the ladder is $y = 2x$. When $x = 2\sqrt{2}$, $y = 2x = 2 \times 2\sqrt{2} = 4\sqrt{2} \Rightarrow$ window is $4\sqrt{2}$m high.

(d) When they have travelled half of the way down the ladder, their point must have an x-coordinate of $\sqrt{2}$. Hence the equation of the vertical line through which they fall is $x = \sqrt{2}$

4. (a) $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 6)$, so $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{3-1} = \frac{4}{2} = 2 \Rightarrow$
$y = 2x + c$. Now $(3, 6)$ on line $\Rightarrow$ $6 = 2 \times 3 + c \Rightarrow$ $c = 0 \Rightarrow$ $y = 2x$

(b) $(x_1, y_1) = (12, 0)$ and $(x_2, y_2) = (3, 6)$, so $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{3-12} = \frac{6}{-9} = -\frac{2}{3} \Rightarrow y = -\frac{2}{3}x + c$. Now
$(12, 0)$ on line $\Rightarrow$ $0 = -\frac{2}{3} \times 12 + c \Rightarrow$ $0 = -8 + c \Rightarrow$ $c = 8$
Hence $y = -\frac{2}{3}x + 8$

(c) First, Stephanie's distance: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 6)$, so
$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

Torpedo distance: $(x_1, y_1) = (12, 0)$ and $(x_2, y_2) = (3, 6)$, so
$d = \sqrt{(12-3)^2 + (0-6)^2} = \sqrt{81 + 36} = \sqrt{117} = 3\sqrt{13}$

Stephanie travels $2\sqrt{5}$m at $\sqrt{5}$ metres/second $\Rightarrow$ 2 seconds.

Torpedo travels $3\sqrt{13}$ at $3\sqrt{13}$ m/sec $\Rightarrow$ 1 second.
$\Rightarrow$ Fire torpedo at time $t = 1$