Assignment 5 Solutions

1. Solve the following simultaneous equations:

a) $2x + 3y = 19$ (1)

$3x - 2y = -4$ (2)

(1) × 2 $4x + 6y = 38$ (3)

(2) × 3 $9x - 6y = -12$ (4)

(3) + (4) $13x = 26, x = 2$

Substitute $x = 2$ into (1), $2 \times 2 + 3y = 19$

$$3y = 15$$

$$y = 5$$

So the solution is (2, 5)

b) $2x - 4y = 4$ (1)

$-x + 2y = -2$ (2)

(2) × 2 $-2x + 4y = -4$ (3)

(2) + (3) $0 = 0$ which is true, so the lines are the same.

2. If $f(x) = x - x^2$ and $g(x) = x + 2$

a) $f(3) = 3 - 3^2 = 3 - 9 = -6$

b) $g(-4) = -4 + 2 = -2$

c) $g(0) = 0 + 2 = 2$, so $f(2) = 2 - 2^2 = 2 - 4 = -2$

d) $f(x + 2) = x + 2 - (x + 2)^2 = x + 2 - (x^2 + 4x + 4) = -x^2 - 3x - 2$

e) $g(x - x^2) = x - x^2 + 2$

3. (1) $f(z) = 5z^2 + 6z - 7$, so

$f(-4) = 5 \times (-4)^2 + 6 \times (-4) - 7 = 80 - 24 - 7 = 49$

(2) $-2y(-2y - 3) = 0$, so

$-2y = 0$ *or* $-2y - 3 = 0$

$y = 0$ $\quad -2y = 3$

$$y = -\frac{3}{2}$$

(3) $-2y^2 + 12y - 10 = 0$, so we use $a = -2, b = 12, c = -10$ in the quadratic formula. Hence

$$\begin{aligned} y &= \frac{-12 \pm \sqrt{12^2 - 4 \times (-2) \times (-10)}}{2 \times (-2)} \\ &= \frac{-12 \pm \sqrt{144 - 80}}{-4} \\ &= \frac{-12 \pm \sqrt{64}}{-4} \\ &= \frac{-12 + 8}{-4} \ \textit{or} \ \frac{-12 - 8}{-4} \\ &= \frac{-4}{-4} \ \textit{or} \ \frac{-20}{-4} \\ &= 1 \ \textit{or} \ 5 \end{aligned}$$

(4) To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

i. $4x(-7 - 3x) = 0$, so

$$\begin{aligned} 4x &= 0 & \textit{or} \qquad -7 - 3x &= 0 \\ x &= 0 & -3x &= 7 \\ & & x &= -\frac{7}{3} \end{aligned}$$

ii. $(-5y + 5)(-6 + 8y) = 0$, so

$$\begin{aligned} -5y + 5 &= 0 & \textit{or} \qquad -6 + 8y &= 0 \\ -5y &= -5 & 8y &= 6 \\ y &= \frac{-5}{-5} & y &= \frac{6}{8} \\ y &= 1 & y &= \frac{3}{4} \end{aligned}$$

iii. $(-9 - 6z)(-2z + 3) = 0$, so

$$\begin{aligned} -9 - 6z &= 0 & \textit{or} \qquad -2z + 3 &= 0 \\ -6z &= 9 & -2z &= -3 \\ z &= \frac{9}{-6} & z &= \frac{3}{2} \\ z &= -\frac{3}{2} & & \end{aligned}$$

iv. $(-4x + 4)^1 = 0$, so $-4x + 4 = 0$, so $-4x = -4$, so $x = \frac{-4}{-4}$, so $x = 1$

(5) $(-4)^0 = 1$

(6) $f(z) = -7(|z|)^2$

When determining the domain of this function, we need to keep in mind the following:

- we can square any number;
- we can find the absolute value of any number.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of z can be substituted into f .

(7) $f(x) = 10(\sqrt{x})^2$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- square root is always positive or 0, so $\sqrt{x} \geq 0$;
- squaring always gives a positive or 0, so $(\sqrt{x})^2 \geq 0$.

Hence, the range of this function is $[0, \infty)$.

(8) $f(z) = \dfrac{-7}{|z| - 3}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $|z| - 3 \neq 0$;
- so $|z| \neq 3$;
- we can find the absolute value of any number, it will always give as a positive or 0, so $z \neq \pm 3$.

Hence, the domain of this function is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$, i.e. $z \neq \pm 3$.

4. Letting p be the cost of a pony ride and c the cost of a camel ride, $3p + 2c = 8.5$ and $2p + 3c = 9$

$3p + 2c = 8.5$ (1)

$2p + 3c = 9$ (2)

(1) × 2 $6p + 4c = 17$ (3)

(2) × –3 $-6p - 9c = -27$ (4)

(3) + (4) $-5c = -10$, so $c = 2$

Substitute $c = 2$ into (1), $3p + 2 \times 2 = 8.5$

$3p = 4.5$

$p = 1.5$

So a pony ride costs $1.50 and a camel ride costs $2. (Check your answer by substituting into (2).)

5. (a) Two points on Marvin's ride are (2, 7) and (1, 1). So m = $\frac{y-y}{x-x} = \frac{7-1}{2-1} = 6$

Hence $y = 6x + c$. Now (1, 1) is on the line, so $1 = 6 \times 1 + c$. So c = –5 and $y = 6x - 5$.

(b) If Charlie's ride is parallel to Marvin's, then the slope of Charlie's ride must also be $m = 6$. $y = 6x + c$. Now Charlie starts at (2, 5), so $5 = 6 \times 2 + c$. So c = –7 and $y = 6x - 7$.

To find the crossing point, use two equations: $y = 6x - 7$ (Charlie) and $y = 3x - 2$ (train). As both the LHSs are the same (y), the RHSs must also be the same. So $6x - 7 = 3x - 2$, so $3x = 5$, $x = \frac{5}{3}$.

Therefore $y = 6 \times \frac{5}{3} - 7$, $y = 3$. Charlie crosses the railway track at $(\frac{5}{3}, 3)$