Assignment 5 Solutions

1. Solve the following simultaneous equations:

   a) \[ 2x + 3y = 19 \] \hspace{1cm} (1)  \\
   \[ 3x - 2y = -4 \] \hspace{1cm} (2)  \\

   \[ (1) \times 2 \quad 4x + 6y = 38 \] \hspace{1cm} (3)  \\
   \[ (2) \times 3 \quad 9x - 6y = -12 \] \hspace{1cm} (4)  \\

   \[ (3) + (4) \quad 13x = 26, \; x = 2 \]  \\
   Substitute \( x = 2 \) into (1), \( 2 \times 2 + 3y = 19 \)  \\
   \[ 3y = 15 \]  \\
   \[ y = 5 \]  \\

   So the solution is \( (2, 5) \)  \\

   b) \[ 2x - 4y = 4 \] \hspace{1cm} (1)  \\
   \[ -x + 2y = -2 \] \hspace{1cm} (2)  \\

   \[ (2) \times 2 \quad -2x + 4y = -4 \] \hspace{1cm} (3)  \\

   \[ (2) + (3) \quad 0 = 0 \] which is true, so the lines are the same.  

2. If \( f(x) = x - x^2 \) and \( g(x) = x + 2 \)

   a) \( f(3) = 3 - 3^2 = 3 - 9 = -6 \)  

   b) \( g(-4) = -4 + 2 = -2 \)  

   c) \( g(0) = 0 + 2 = 2 \), so \( f(2) = 2 - 2^2 = 2 - 4 = -2 \)  

   d) \( f(x + 2) = x + 2 - (x + 2)^2 = x + 2 - (x^2 + 4x + 4) = -x^2 - 3x - 2 \)  

   e) \( g(x - x^2) = x - x^2 + 2 \)  

3. \( f(x) = 5x^2 + 6x - 7 \), so

   \( f(-4) = 5 \times (-4)^2 + 6 \times (-4) - 7 = 80 - 24 - 7 = 49 \)  

   \( -2y (-2y - 3) = 0 \), so 

   \[ -2y = 0 \quad \text{or} \quad -2y - 3 = 0 \]  

   \[ y = 0 \quad \text{or} \quad -2y = 3 \]  

   \[ y = -\frac{3}{2} \]
(3) \(-2y^2 + 12y - 10 = 0\), so we use \(a = -2, b = 12, c = -10\) in the quadratic formula. Hence
\[
y = \frac{-12 \pm \sqrt{12^2 - 4 \times (-2) \times (-10)}}{2 \times (-2)}
\]
\[
= \frac{-12 \pm \sqrt{144 - 80}}{-4}
\]
\[
= \frac{-12 \pm \sqrt{64}}{-4}
\]
\[
= \frac{-12 + 8}{-4} \quad \text{or} \quad \frac{-12 - 8}{-4}
\]
\[
= \frac{-4}{-4} \quad \text{or} \quad \frac{-20}{-4}
\]
\[
= 1 \quad \text{or} \quad 5
\]

(4) To solve each of these, remember that if \(a \times b = 0\), then either \(a = 0\) or \(b = 0\); and also that \(0^n = 0\) for any natural number \(n\). Then:

i. \(4x (-7 - 3x) = 0\), so
\[
\begin{align*}
4x &= 0 & \text{or} & & -7 - 3x &= 0 \\
x &= 0 & & -3x &= 7 \\
& & & x &= -\frac{7}{3}
\end{align*}
\]

ii. \((-5y + 5) (-6 + 8y) = 0\), so
\[
\begin{align*}
-5y + 5 &= 0 & \text{or} & & -6 + 8y &= 0 \\
-5y &= -5 & & 8y &= 6 \\
y &= -1 & & y &= \frac{3}{4}
\end{align*}
\]

iii. \((-9 - 6x) (-2x + 3) = 0\), so
\[
\begin{align*}
-9 - 6x &= 0 & \text{or} & & -2x + 3 &= 0 \\
-6x &= 9 & & -2x &= -3 \\
z &= -\frac{9}{6} & & z &= \frac{3}{2}
\end{align*}
\]

iv. \((-4x + 4)^1 = 0\), so \(-4x + 4 = 0\), so \(-4x = -4\), so \(x = \frac{-4}{-4}\), so \(x = 1\)

(5) \((-4)^0 = 1\)

(6) \(f(x) = -7 \cdot (|x|)^2\)

When determining the domain of this function, we need to keep in mind the following:

- we can square any number;
- we can find the absolute value of any number.

Hence, the domain of this function is \((-\infty, \infty)\), i.e. any value of \(x\) can be substituted into \(f\).

(7) \(f(x) = 10 \cdot (\sqrt{x})^2\)

When evaluating the range, we need to keep in mind the following (starting with variable \(x\)):

- square root is always positive or 0, so \(\sqrt{x} \geq 0\);
- squaring always gives a positive or 0, so \((\sqrt{x})^2 \geq 0\).

Hence, the range of this function is \([0, \infty)\).

(8) \(f(x) = \frac{-7}{|x| - 3}\)

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so \(|x| - 3 \neq 0\);
- \(so \ |x| \neq 3;\)
- we can find the absolute value of any number, it will always give as a positive or 0, so \(x \neq \pm 3\).

Hence, the domain of this function is \((-\infty, -3) \cup (-3, 3) \cup (3, \infty)\), i.e. \(x \neq \pm 3\).
4. Letting $p$ be the cost of a pony ride and $c$ the cost of a camel ride, $3p + 2c = 8.5$ and $2p + 3c = 9$

$3p + 2c = 8.5 \quad (1)$

$2p + 3c = 9 \quad (2)$

$(1) \times 2 \quad 6p + 4c = 17 \quad (3)$

$(2) \times -3 \quad -6p - 9c = -27 \quad (4)$

$(3) + (4) \quad -5c = -10$, so $c = 2$

Substitute $c = 2$ into $(1), 3p + 2 \times 2 = 8.5$

$3p = 4.5$

$p = 1.5$

So a pony ride costs $1.50 and a camel ride costs $2. (Check your answer by substituting into (2).)

5. (a) Two points on Marvin’s ride are (2, 7) and (1, 1). So $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 1}{2 - 1} = 6$

Hence $y = 6x + c$. Now (1, 1) is on the line, so $1 = 6 \times 1 + c$. So $c = -5$ and $y = 6x - 5$.

(b) If Charlie’s ride is parallel to Marvin’s, then the slope of Charlie’s ride must also be $m = 6$.

$y = 6x + c$. Now Charlie starts at (2, 5), so $5 = 6 \times 2 + c$. So $c = -7$ and $y = 6x - 7$.

To find the crossing point, use two equations: $y = 6x - 7$ (Charlie) and $y = 3x - 2$ (train). As both the LHSs are the same ($y$), the RHSs must also be the same. So $6x - 7 = 3x - 2$, so $3x = 5$, $x = \frac{5}{3}$.

Therefore $y = 6 \times \frac{5}{3} - 7$, $y = 3$. Charlie crosses the railway track at $\left(\frac{5}{3}, 3\right)$.