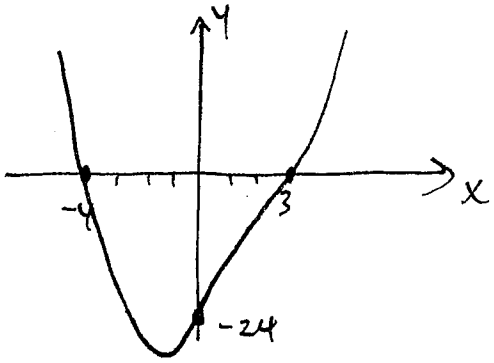
Assignment 6 Solutions

1. (1) $2z^2 + 2z - 24 = 0$, so we use $a = 2, b = 2, c = -24$ in the quadratic formula. Hence

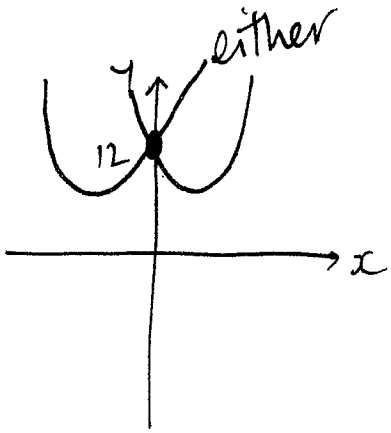
$$\begin{aligned} z &= \frac{-2 \pm \sqrt{2^2 - 4 \times 2 \times (-24)}}{2 \times 2} \\ &= \frac{-2 \pm \sqrt{4 - (-192)}}{4} \\ &= \frac{-2 \pm \sqrt{196}}{4} \\ &= \frac{-2 + 14}{4} \text{ or } \frac{-2 - 14}{4} \\ &= \frac{12}{4} \text{ or } \frac{-16}{4} \\ &= 3 \text{ or } -4 \end{aligned}$$



(2) $3x^2 + 11x + 12 = 0$, so we use $a = 3, b = 11, c = 12$ in the quadratic formula. Hence

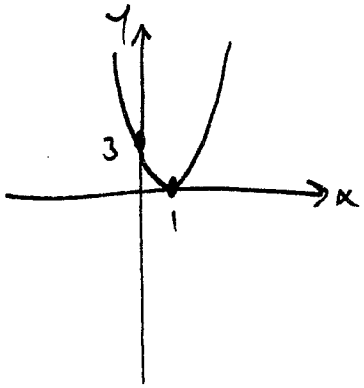
$$\begin{aligned} x &= \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times 12}}{2 \times 3} \\ &= \frac{-11 \pm \sqrt{121 - 144}}{6} \\ &= \frac{-11 \pm \sqrt{-23}}{6} \end{aligned}$$

Hence there is no solution.



(3) $3x^2 - 6x + 3 = 0$, so we use $a = 3, b = -6, c = 3$ in the quadratic formula. Hence

$$\begin{aligned} x &= \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 3}}{2 \times 3} \\ &= \frac{6 \pm \sqrt{36 - 36}}{6} \\ &= \frac{6 \pm \sqrt{0}}{6} \\ &= \frac{6}{6} \\ &= 1 \end{aligned}$$



2. Let P be the amount invested, r be the interest rate per time period, n be the number of time periods and F be the final value. In each case, $P = 400$. Then:

   i. Interest compounds annually, so we use the rate and number of time periods given in the question.
   Hence $r = 5.0\% = 0.05$ and $n = 5$, so $F = 400 \times (1 + 0.05)^5 = 400 \times 1.05^5 \approx 510.51$.
   The final balance is \$510.51.

   ii. Interest compounds twice a year, so we need to halve the rate and double the number of time periods given in the question.
   Hence $r = 2.5\% = 0.025$ and $n = 10$, so $F = 400 \times (1 + 0.025)^{10} = 400 \times 1.025^{10} \approx 512.03$.
   The final balance is \$512.03.

   iii. Interest compounds 4 times a year, so we need to divide the given rate by 4 and multiply the given number of years by 4.
   Hence $r = 1.3\% = 0.0125$ and $n = 20$, so $F = 400 \times (1 + 0.0125)^{20} = 400 \times 1.0125^{20} \approx 512.81$.
   The final balance is \$512.81.

   iv. Interest compounds 12 times a year, so we need to divide the given rate by 12 and multiply the given number of years by 12.
   Hence $r = 0.4\% = 0.0042$ and $n = 60$, so $F = 400 \times (1 + 0.0042)^{60} = 400 \times 1.0042^{60} \approx 513.34$.
   The final balance is \$513.34.

   v. Interest compounds continuously, so $F = 400e^{0.05 \times 5} = 400e^{0.25} \approx 513.61$.
   The final balance is \$513.61.

3. i. $\log_{16} 16^{18} = 18$

ii. $27 = 3^3$, so $\log_3 27 = 3$

iii. $\frac{1}{9} = 3^{-2}$, so $\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$. Hence the answer is -2.

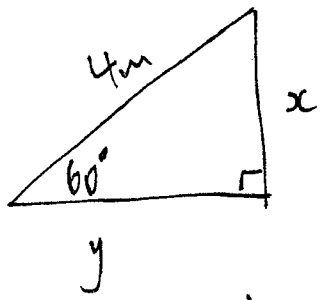
iv. $1000 = 10^3$, so $\log_{10} 1000 = 3$

v. $\frac{1}{10000} = 10^{-4}$, so $\log_{10} \frac{1}{10000} = -4$

vi. $\ln e^{12} = 12$

vii. $\frac{1}{e^{18}} = e^{-18}$, so $\ln \frac{1}{e^{18}} = \ln e^{-18} = -18$. Hence the answer is -18.

4.



let x = height ladder reaches on wall

y = distance of ladder from base of wall

a) $\sin 60° = \frac{\text{opp}}{\text{hyp}}$

$\sin 60° = \frac{x}{4}$

$\therefore x = 4 \sin 60$

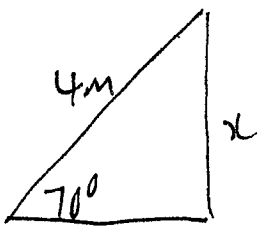
$\approx 3.5\text{m}$

b) $\cos 60 = \frac{\text{adj}}{\text{hyp}} = \frac{y}{4}$

$\therefore y = 4 \times \cos 60°$

$= 2\text{m}$

c)



$\sin 70° = \frac{\text{opp}}{\text{hyp}}$

$\sin 70° = \frac{x}{4}$

$\therefore x = 4 \sin 70°$

$\approx 3.76\text{m}$

$\therefore$ The ladder will not reach.

5.

i. $0 = -11x + 6$, so $11x = 6$, so $x = \frac{6}{11}$. Hence this is a vertical line, with x positive. Hence the matching graph is Graph B.

ii. $2y + 8x^2 - 15 = -y + 13x^2 - 16$, so $3y = 5x^2 - 1$. This equation includes an x^2 term with a positive coefficient, so the graph is a parabola which turns upwards. Also, the y-intercept is negative. Hence the matching graph is Graph Q.

iii. $y = e^{5x}$, which is a graph of exponential growth. Hence the matching graph is Graph K.

iv. $-10y - x + 2 = 16y + 14$, so $26y = -x - 12$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

v. $-x + 3 = 8y - 11x + 16$, so $8y = 10x - 13$. Hence this is a straight line, with positive gradient and negative y-intercept. Hence the matching graph is Graph E.

vi. $-10y + 10 = 14y + 6x^2 + 10$, so $24y = -6x^2$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is 0. Hence the matching graph is Graph S.

vii. $-12y - 9x^2 + 8 = -9y + 7x^2 - 1$, so $3y = -16x^2 + 9$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is positive. Hence the matching graph is Graph R.

viii. $-12x - 5 = 2$, so $-12x = 7$, so $x = -\frac{7}{12}$. Hence this is a vertical line, with x negative. Hence the matching graph is Graph A.

3.