



THE UNIVERSITY
OF QUEENSLAND

VENUE:

SEAT NUMBER:

STUDENT NUMBER:

FIRST NAME:

LAST NAME:

FINAL EXAMINATION

First Semester, 2007

St Lucia Campus

MATH1050 – Mathematical Foundations

PERUSAL TIME 10 mins. No writing during perusal time.

WRITING TIME 2 hours

EXAMINER Dr Jon Links, Dr. Katrina Hibberd

NO. OF PAGES (*include title page and attachments*) **21 Single Sided**

Exam Type: Closed Book - Specified materials permitted

Permitted Materials: Calculator - Yes - Non-programmable calculators only
Dictionary - Yes - Bilingual Dictionary only
Other – No electronic aids are permitted (e.g. laptops, phone)

Answer: On examination paper in spaces provided
Use the back of the pages if necessary.

Number of Questions: 30

Weighting/Marks: 50%/ 50 marks

Special Instructions: Students must comply with the General Award Rules 1A.7 and 1A.8
which outline the responsibilities of students during an examination.

**THIS EXAMINATION PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM**

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First Semester Examination, 2007 (continued)

EXAMINER'S USE ONLY			
QUESTION	MARK	QUESTION	MARK
1-20		26	
21		27	
22		28	
23		29	
24		30	
25		Total	

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For each of the following 20 multiple choice questions, write the letter corresponding to the correct answer in the space provided at the end of the question. Each correct answer is worth 0.5 marks, each incorrect answer is worth 0 marks.

1. Which of the following statements is true ? _____
 - (a) $\sin\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$
 - (b) $\sin\left(\theta - \frac{\pi}{2}\right) = \cos(\theta)$
 - (c) $\sin\left(\theta - \frac{\pi}{2}\right) = \tan(\theta)$
 - (d) none of the above
2. If $|x| < 2$ which of the following statements is true? _____
 - (a) $-2 \leq x < 2$
 - (b) $-2 < x < 2$
 - (c) $-2 \leq x \leq 2$
 - (d) none of the above
3. Which of the following intervals corresponds to the inequality $|-x| > 5$? _____
 - (a) $(-5, 5)$
 - (b) $[-5, 5]$
 - (c) $(-\infty, -5) \cup (5, \infty)$
 - (d) $(-\infty, -5] \cup [5, \infty)$
4. The domain of the function $f(x) = 1 + \cos(x^2)$ is: _____
 - (a) $(-\infty, \infty)$
 - (b) $(0, \infty)$
 - (c) $[0, \infty)$
 - (d) none of the above
5. The range of the function $f(x) = 1 + \cos(x^2)$ is: _____
 - (a) $(-\infty, \infty)$
 - (b) $[-2, 0]$
 - (c) $[0, 2]$
 - (d) $[-2, 2]$

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6. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be any vectors in $\mathbb{R}^3$. Which of the following statements is generally false? _____

- (a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
- (b) $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$
- (c) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$
- (d) none of the above

7. If $S = 2x^2 - 4x^4 + 6x^6 - 8x^8$, which of the following statements is true? _____

- (a) $S = 2 \sum_{k=0}^3 (-1)^k (k+1) x^{2k+2}$
- (b) $S = 2 \sum_{k=1}^4 (-1)^{(k+1)} (k+1) x^{2k}$
- (c) $S = 2 \sum_{k=0}^3 (-1)^{(k+1)} (k+1) x^{2k+2}$
- (d) none of the above

8. If A and B are invertible $n \times n$ matrices and $C = AB$, which of the following statements is false? _____

- (a) C is invertible
- (b) C is singular
- (c) C non-singular
- (d) none of the above.

9. If $y = x^4 + x^3 + 2x^2 + 4x$ then $\frac{dy}{dx}$ is: _____

- (a) $4x^3 + 3x^2 + 4x + 4$
- (b) $4x^3 + 3x^2 + 4x$
- (c) $4x^3 + 3x^2$
- (d) $4x^3$

10. If $y = x^3 + 2x^2 + 4x$ then $\int y \, dx$ is: _____

- (a) $\frac{1}{4}x^4 + \frac{4}{3}x^3 + x^2 + c$
- (b) $x^4 + \frac{4}{3}x^3 + 2x^2 + x + c$
- (c) $x^4 + \frac{2}{3}x^3 + x^2 + x + c$
- (d) $\frac{1}{4}x^4 + \frac{2}{3}x^3 + 2x^2 + c$

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11. If A, B are $n \times n$ matrices, which of the following is always true? ____
- (a) $AB = BA$
 - (b) $(AB)^T = A^T B^T$
 - (c) $(AB)^T = B^T A^T$
 - (d) none of the above
12. If A, B are invertible $n \times n$ matrices, which of the following is always true? ____
- (a) $(AB)^{-1} = A^{-1} B^{-1}$
 - (b) $(AB)^{-1} = B^{-1} A^{-1}$
 - (c) $(A + B)^{-1} = A^{-1} + B^{-1}$
 - (d) none of the above
13. If $y = e^{(x^2-x)}$, which of the following is true? ____
- (a) $\frac{dy}{dx} = (2x - 1)e^{(x^2-x)}$
 - (b) $\frac{dy}{dx} = e^{(2x-1)}$
 - (c) $\frac{dy}{dx} = (2x - 1)e^{(2x-1)}$
 - (d) none of the above
14. If A is an $n \times m$ matrix and B is an $m \times n$ matrix, which of the following expressions is defined? ____
- (a) $A + B$
 - (b) $A + B^T$
 - (c) $A^T + B^T$
 - (d) $A^T B$
15. If A is an $n \times m$ matrix and B is an $m \times n$ matrix, which of the following expressions is not defined? ____
- (a) $A + B$
 - (b) $A + B^T$
 - (c) $A^T + B$
 - (d) AB

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16. If $f = x^2$ and $g = x + 2$, which of the following is true? _____
- (a) $(f \circ g)(x) = x^2 + 2$
 - (b) $(f \circ g)(x) = x^2 + x + 2$
 - (c) $(f \circ g)(x) = (x + 2)^2 + x + 2$
 - (d) $(f \circ g)(x) = (x + 2)^2$
17. If $f = x^2$ and $g = x + 2$, which of the following is true? _____
- (a) $(g \circ f)(x) = x^2 + 2$
 - (b) $(g \circ f)(x) = x^2 + x + 2$
 - (c) $(g \circ f)(x) = (x + 2)^2 + x + 2$
 - (d) $(g \circ f)(x) = (x + 2)^2$
18. If z is a complex number, which of the following statements is true? _____
- (a) $z = \bar{z}$
 - (b) $z\bar{z} = 1$
 - (c) $z\bar{z} = |z|^2$
 - (d) none of the above
19. If y and z complex numbers, which of the following is false? _____
- (a) $yz = zy$
 - (b) $\overline{y + z} = \bar{y} + \bar{z}$
 - (c) $\overline{y - z} = \bar{y} - \bar{z}$
 - (d) $y\bar{z} = \bar{y}z$
20. If y and z are complex numbers, which of the following is true? _____
- (a) $|y + z| = |y| + |z|$
 - (b) $|y - z| = |y| - |z|$
 - (c) $|yz| = |y| \cdot |z|$
 - (d) none of the above

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21. Registration fees for the annual Conference of the Australian Philosophical Society are \$110 full price and \$60 for students. The total number of people attending is 550, and the takings from registration fees is \$49,500. Set up a pair of simultaneous equations and solve them using matrices in order to determine how many students attended. (3 marks)

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22. Use mathematical induction to prove that $f(n) = 11^n - 6$ is divisible by 5 for all $n \in \mathbb{N}$. (3 marks)

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23. This question is about proving the Sine Law and the Cosine Law, with reference to the figure below.

- (a) Find two expressions for the length d , one in terms of the angle A and one in terms of the angle B . Hence deduce that the Sine Law

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

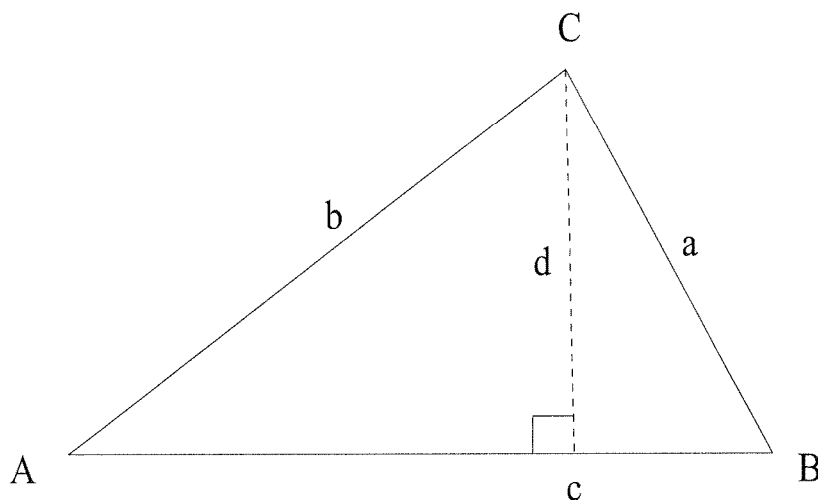
holds. (1.5 marks)

- (b) Choosing the point A to be the origin, the co-ordinates of the point C are $(b \cos A, b \sin A)$, while the co-ordinates of the point B are $(c, 0)$. In terms of these coordinates, calculate the magnitude of $\overrightarrow{BC}$. (1.5 marks)

- (c) From the results of (b), deduce that the Cosine Law

$$a^2 = b^2 + c^2 - 2bc \cos A$$

holds. (1 mark)



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23. continued

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24. The first three terms of a geometric sequence are 0.25, 0.5, 1.
- (a) What is the common ratio of the sequence? (0.5 mark)
 - (b) What is the 6th term in the sequence. (1 mark)
 - (c) For which value of n does the associated series S_n first exceed 100.
(1.5 marks)
 - (d) Does the series S_n converge or diverge as $n \rightarrow \infty$? (1 mark)

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25. (a) Evaluate the derivative of the function

$$f(x) = (4x - 1)e^{(4x^2 - 2x)}. \quad (2 \text{ marks})$$

- (b) Evaluate the indefinite integral

$$\int (4x - 1)e^{(4x^2 - 2x)} dx. \quad (2 \text{ marks})$$

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26. Solve the following inequality. Write your solution in interval notation.

$$\frac{2y - 1}{y - 1} < 4.$$

(3 marks)

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27. Let $f(x) = \frac{1}{\sqrt{2x-1}}$ and let $g(x) = x^2$.

- (a) State the domain and range of f . (1 mark)
- (b) State the domain and range of g . (1 mark)
- (c) Determine $(g \circ f)(x)$ and state its domain and range. (2 marks)
- (d) Is g a one-to-one function? Explain your answer. (1 mark)

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28. Let $f(x) = \frac{x+1}{x-1}$.

- (a) Use the definition of the derivative as a limit to determine the slope of the tangent line at $x = 0$. (3 marks)
- (b) Find the equation of the tangent line to f at $x = 0$. (2 marks)

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28. continued

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29. Let $f(x) = 2x^2 + 3x$.

- (a) Evaluate the Riemann sum $A_L = \sum_{i=1}^n f(x_{i-1})\Delta x$ for $f(x)$, with $1 \leq x \leq 2$, having 10 subintervals. (3 marks)
- (b) Evaluate the definite integral $\int_0^1 f(x) dx$. (1 mark)

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29. continued

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30. (a) Let $z = 1 + i$. Use De Moivre's Theorem to evaluate z^4 . (2 marks)
- (b) One solution of $z^3 - 3z^2 + 4z - 2 = 0$ is $z = 1 + i$. Find all the other solutions of this equation over $\mathbb{C}$. (3 marks)

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30. continued

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If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\det(A) = ad - bc$.

If $\det(A) \neq 0$, then $A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

If $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ and $\hat{\mathbf{v}} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$.

Scalar product: $\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$.

Vector product: $\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

Cosine Law: $a^2 = b^2 + c^2 - 2bc \cos A$, Sine Law: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Series: $\sum_{j=1}^n j = \frac{n(n+1)}{2}$, $\sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}$.

If $\{a_j\}_{j=1}^{\infty}$ is a geometric sequence with first term a and common ratio r , then

$$S_n = \sum_{j=1}^n a_j = \frac{a(r^n - 1)}{r - 1}.$$

The definition of the derivative $f'(a)$ of a function f at the number a is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Differentiation rules:

$$(fg)' = f'g + fg', \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \quad (f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Trigonometric identities:

$$\begin{aligned} \sin(\theta + \phi) &= \sin(\theta) \cos(\phi) + \cos(\theta) \sin(\phi) \\ \cos(\theta + \phi) &= \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi) \end{aligned}$$

Euler's formula: $e^{i\theta} = \cos \theta + i \sin \theta$.

De Moivre's theorem: If $z = r(\cos(\theta) + i \sin(\theta))$ then $z^n = r^n(\cos(n\theta) + i \sin(n\theta))$.