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THE UNIVERSITY OF QUEENSLAND

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First Semester Examination, 2007

# MATH1050

## Mathematical Foundations

**Time:** **Two hours** for working

Ten minutes for perusal before examination begins

**CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.**

Use the back pages if the space provided is insufficient.
Candidates should attempt ALL questions. All questions carry the indicated number of marks. There are 50 marks in total. Check that this examination paper has 19 printed pages.

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**MATH1050 — Mathematical Foundations**
**First Semester Examination, 2007 (continued)**

| **EXAMINER'S USE ONLY** | | | |
|---|---|---|---|
| QUESTION | MARK | QUESTION | MARK |
| 1-20 | | 26 | |
| 21 | | 27 | |
| 22 | | 28 | |
| 23 | | 29 | |
| 24 | | 30 | |
| 25 | | Total | |

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**MATH1050 — Mathematical Foundations**
**First Semester Examination, 2007 (continued)**

For each of the following 20 multiple choice questions, write the letter corresponding to the correct answer in the space provided at the end of the question. Each correct answer is worth 0.5 marks, each incorrect answer is worth 0 marks.

1. Which of the following statements is true ? *d*

   (a) $\sin\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$

   (b) $\sin\left(\theta - \frac{\pi}{2}\right) = \cos(\theta)$

   (c) $\sin\left(\theta - \frac{\pi}{2}\right) = \tan(\theta)$

   (d) none of the above

2. If $|x| < 2$ which of the following statements is true? *b*

   (a) $-2 \leq x < 2$

   (b) $-2 < x < 2$

   (c) $-2 \leq x \leq 2$

   (d) none of the above

3. Which of the following intervals corresponds to the inequality $|-x| > 5$? *c*

   (a) $(-5, 5)$

   (b) $[-5, 5]$

   (c) $(-\infty, -5) \cup (5, \infty)$

   (d) $(-\infty, -5] \cup [5, \infty)$

4. The domain of the function $f(x) = 1 + \cos(x^2)$ is: *a*

   (a) $(-\infty, \infty)$

   (b) $(0, \infty)$

   (c) $[0, \infty)$

   (d) none of the above

5. The range of the function $f(x) = 1 + \cos(x^2)$ is: *c*

   (a) $(-\infty, \infty)$

   (b) $[-2, 0]$

   (c) $[0, 2]$

   (d) $[-2, 2]$

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**MATH1050 — Mathematical Foundations**
**First Semester Examination, 2007 (continued)**

6. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be any vectors in $\mathbb{R}^3$. Which of the following statements is generally false? *c*

   (a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
   (b) $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$
   (c) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$
   (d) none of the above

7. If $S = 2x^2 - 4x^4 + 6x^6 - 8x^8$, which of the following statements is true? *a*

   (a) $S = 2\sum_{k=0}^{3}(-1)^k(k+1)x^{2k+2}$
   (b) $S = 2\sum_{k=1}^{4}(-1)^{(k+1)}(k+1)x^{2k}$
   (c) $S = 2\sum_{k=0}^{3}(-1)^{(k+1)}(k+1)x^{2k+2}$
   (d) none of the above

8. If $A$ and $B$ are invertible $n \times n$ matrices and $C = AB$, which of the following statements is false? *b*

   (a) $C$ is invertible
   (b) $C$ is singular
   (c) $C$ non-singular
   (d) none of the above.

9. If $y = x^4 + x^3 + 2x^2 + 4x$ then $\dfrac{dy}{dx}$ is: *a*

   (a) $4x^3 + 3x^2 + 4x + 4$
   (b) $4x^3 + 3x^2 + 4x$
   (c) $4x^3 + 3x^2$
   (d) $4x^3$

10. If $y = x^3 + 2x^2 + 4x$ then $\int y\,dx$ is: *d*

    (a) $\dfrac{1}{4}x^4 + \dfrac{4}{3}x^3 + x^2 + c$
    (b) $x^4 + \dfrac{4}{3}x^3 + 2x^2 + x + c$
    (c) $x^4 + \dfrac{2}{3}x^3 + x^2 + x + c$
    (d) $\dfrac{1}{4}x^4 + \dfrac{2}{3}x^3 + 2x^2 + c$

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11. If $A$, $B$ are $n \times n$ matrices, which of the following is always true? _c_

    (a) $AB = BA$
    (b) $(AB)^T = A^T B^T$
    (c) $(AB)^T = B^T A^T$
    (d) none of the above

12. If $A$, $B$ are invertible $n \times n$ matrices, which of the following is always true? ? _b_

    (a) $(AB)^{-1} = A^{-1} B^{-1}$
    (b) $(AB)^{-1} = B^{-1} A^{-1}$
    (c) $(A + B)^{-1} = A^{-1} + B^{-1}$
    (d) none of the above

13. If $y = e^{(x^2 - x)}$, which of the following is true ? _a_

    (a) $\dfrac{dy}{dx} = (2x - 1)e^{(x^2 - x)}$
    (b) $\dfrac{dy}{dx} = e^{(2x-1)}$
    (c) $\dfrac{dy}{dx} = (2x - 1)e^{(2x-1)}$
    (d) none of the above

14. If $A$ is an $n \times m$ matrix and $B$ is an $m \times n$ matrix, which of the following expressions is defined ? _b_

    (a) $A + B$
    (b) $A + B^T$
    (c) $A^T + B^T$
    (d) $A^T B$

15. If $A$ is an $n \times m$ matrix and $B$ is an $m \times n$ matrix, which of the following expressions is not defined ? _a_

    (a) $A + B$
    (b) $A + B^T$
    (c) $A^T + B$
    (d) $AB$

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16. If $f = x^2$ and $g = x + 2$, which of the following is true ? _d_

    (a) $(f \circ g)(x) = x^2 + 2$
    (b) $(f \circ g)(x) = x^2 + x + 2$
    (c) $(f \circ g)(x) = (x + 2)^2 + x + 2$
    (d) $(f \circ g)(x) = (x + 2)^2$

17. If $f = x^2$ and $g = x + 2$, which of the following is true ? _a_

    (a) $(g \circ f)(x) = x^2 + 2$
    (b) $(g \circ f)(x) = x^2 + x + 2$
    (c) $(g \circ f)(x) = (x + 2)^2 + x + 2$
    (d) $(g \circ f)(x) = (x + 2)^2$

18. If $z$ is a complex number, which of the following statements is true ? _c_

    (a) $z = \overline{z}$
    (b) $z\overline{z} = 1$
    (c) $z\overline{z} = |z|^2$
    (d) none of the above

19. If $y$ and $z$ complex numbers, which of the following is false ? _d_

    (a) $yz = zy$
    (b) $\overline{y + z} = \overline{y} + \overline{z}$
    (c) $\overline{y - z} = \overline{y} - \overline{z}$
    (d) $y\overline{z} = \overline{y}z$

20. If $y$ and $z$ are complex numbers, which of the following is true ? _c_

    (a) $|y + z| = |y| + |z|$
    (b) $|y - z| = |y| - |z|$
    (c) $|yz| = |y|.|z|$
    (d) none of the above

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21. Registration fees for the annual Conference of the Australian Philosophical Society are \$110 full price and \$60 for students. The total number of people attending is 550, and the takings from registration fees is \$49,500. Set up a pair of simultaneous equations and solve them using matrices in order to determine how many students attended. (3 marks)

Solution
Let $x$ denote the number of full fee registrations and $y$ the number of student registrations. Then

$$\begin{aligned} x + y &= 550 \\ 110x + 60y &= 49,500 \end{aligned}$$

As a matrix equation we have

$$\begin{pmatrix} 1 & 1 \\ 110 & 60 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 550 \\ 49,500 \end{pmatrix}.$$

(1 mark)

$$\begin{aligned} \begin{vmatrix} 1 & 1 \\ 110 & 60 \end{vmatrix} &= 60 - 110 = -50 \\ \begin{pmatrix} 1 & 1 \\ 110 & 60 \end{pmatrix}^{-1} &= \frac{-1}{50} \begin{pmatrix} 60 & -1 \\ -110 & 1 \end{pmatrix}. \end{aligned}$$

(1 mark)
Hence

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \frac{-1}{50} \begin{pmatrix} 60 & -1 \\ -110 & 1 \end{pmatrix} \begin{pmatrix} 550 \\ 49,500 \end{pmatrix} \\ &= \frac{-1}{50} \begin{pmatrix} 60 \times 550 - 49,500 \\ -110 \times 550 + 49,500 \end{pmatrix} \\ &= \begin{pmatrix} 330 \\ 220 \end{pmatrix}. \end{aligned}$$

There were 220 students attending the conference. (1 mark)

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22. Use induction to prove that $f(n) = 11^n - 6$ is divisible by 5 for all $n \in \mathbb{N}$.
(3 marks)
Solution
When $n = 1$, $f(1) = 11 - 6 = 5$ which is divisible by 5 so the result is true for $n = 1$. (1 mark)

Now

$$\begin{aligned} f(n+1) &= 11^{n+1} - 6 \\ &= 11 \times 11^n - 6 \\ &= (11 \times 11^n - 66) + (66 - 6) \\ &= 11(11^n - 6) + 60 \\ &= 11f(n) + 60. \qquad (1 \text{ mark}) \end{aligned}$$

Assuming the result is true for $n$ we can write

$$\frac{f(n)}{5} = X$$

for some $X \in \mathbb{N}$. Now

$$\begin{aligned} \frac{f(n+1)}{5} &= \frac{11f(n) + 60}{5} \\ &= \frac{11f(n)}{5} + \frac{60}{5} \\ &= 11X + 12 \in \mathbb{N}. \end{aligned}$$

so the result is true for $n + 1$. Since we have already shown the result holds for $n = 1$, by induction, $f(n) = 11^n - 6$ is divisible by 5 for all $n \in \mathbb{N}$. (1 mark)

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23. This question is about proving the Sine Law and the Cosine Law, with reference to the figure below.

    (a) Find two expressions for the length $d$, one in terms of the angle $A$ and one in terms of the angle $B$. Hence deduce that the Sine Law

    $$\frac{\sin A}{a} = \frac{\sin B}{b}$$

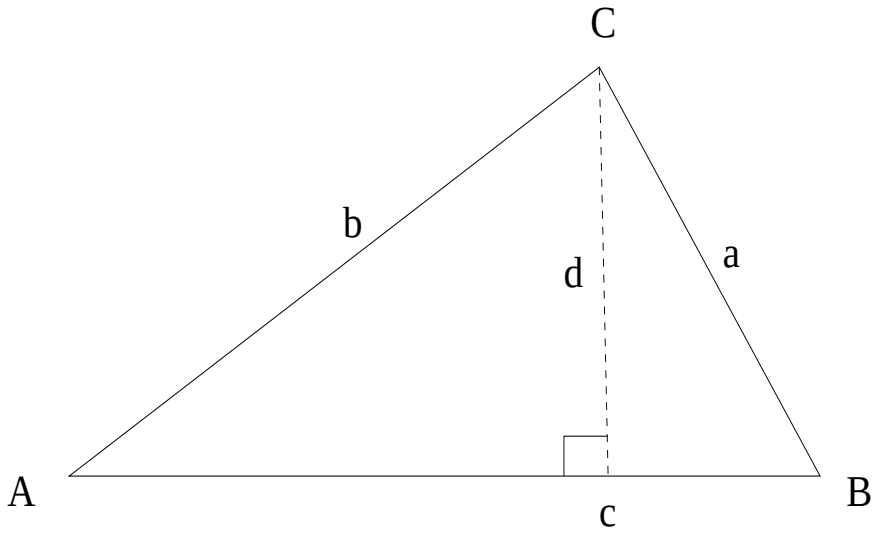
    holds. (1.5 marks)

    (b) Choosing the point $A$ to be the origin, the co-ordinates of the point $C$ are $(b\cos A, b\sin A)$, while the co-ordinates of the point $B$ are $(c, 0)$. In terms of these cordinates, calculated the magnitude of $\overrightarrow{BC}$. (1.5 marks)

    (c) From the results of (b), deduce that the Cosine Law

    $$a^2 = b^2 + c^2 - 2bc\cos A$$

    holds. (1 mark)




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Solution

(a) From the diagram,

$$\begin{aligned} \sin A &= \frac{d}{b} \implies d = b\sin A, \qquad (0.5 \text{ mark}) \\ \sin B &= \frac{d}{a} \implies d = a\sin B \qquad (0.5 \text{ mark}) \end{aligned}$$

Then

$$\begin{aligned} b\sin A &= a\sin B \\ \frac{\sin A}{a} &= \frac{\sin B}{b}. \qquad (0.5 \text{ mark}) \end{aligned}$$

(b)

$$\begin{aligned} \overrightarrow{BC} &= (b\cos A,\, b\sin A) - (c,\, 0) \\ &= (b\cos A - c,\, b\sin A). \qquad (0.5 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \left|\left|\overrightarrow{BC}\right|\right| &= \sqrt{(b\cos A - c)^2 + (b\sin A)^2} \\ &= \sqrt{b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A} \\ &= \sqrt{b^2 + c^2 - 2bc\cos A}. \qquad (1 \text{ mark}) \end{aligned}$$

(c)

$$\begin{aligned} a &= \left|\left|\overrightarrow{BC}\right|\right| \\ a^2 &= \left|\left|\overrightarrow{BC}\right|\right|^2 \\ &= b^2 + c^2 - 2bc\cos A. \qquad (1 \text{ mark}) \end{aligned}$$

24. The first three terms of a geometric sequence are 0.25, 0.5, 1.

    (a) What is the common ratio of the sequence? (0.5 mark)

    (b) What is the 6th term in the sequence. (1 mark)

    (c) For which value of $n$ does the associated series $S_n$ first exceed 100. (1.5 marks)

    (d) Does the series $S_n$ converge or diverge as $n \to \infty$? (1 mark)

Solution

(a) The common ratio is $r = 2$. (0.5 mark)

(b) $a_n = ar^{n-1}$ (0.5 mark), so $a_6 = 0.25 \times 2^5 = 8$. (0.5 mark)

(c)

$$\begin{aligned} S_n &= \frac{a(r^n - 1)}{r - 1} \\ 100 &= 0.25 \times \frac{2^n - 1}{2 - 1} = 0.25 \times (2^n - 1) \qquad \text{(0.5 mark)} \\ 400 &= 2^n - 1 \\ 2^n &= 401 \\ n &= \frac{\ln 401}{\ln 2} \qquad \text{(0.5 mark)} \\ &= 8.65 \end{aligned}$$

The series first exceeds 100 when $n = 9$. (0.5 mark)

(d) The series diverges since $r > 1$. (1 mark)

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25. (a) Evaluate the derivative of the function

$$f(x) = (4x-1)e^{(4x^2-2x)}. \qquad \text{(2 marks)}$$

(b) Evaluate the indefinite integral

$$\int (4x-1)e^{(4x^2-2x)}\, dx. \qquad \text{(2 marks)}$$

Solution

(a) Need to use both the chain rule and product rule. Put $u = 4x-1$, $v = e^w$ with $w = 4x^2 - 2x$ so

$$\begin{aligned} \frac{du}{dx} &= 4 \\ \frac{dv}{dw} &= e^w = e^{(4x^2-2x)} \\ \frac{dw}{dx} &= 8x-2. \qquad \text{(1 mark)} \end{aligned}$$

Now

$$\begin{aligned} f'(x) &= \frac{du}{dx}.v + u.\frac{dv}{dx} \\ &= \frac{du}{dx}.v + u.\frac{dv}{dw}.\frac{dw}{dx} \\ &= 4e^{(4x^2-2x)} + (4x-1).e^{(4x^2-2x)}.(8x-2) \\ &= (4 + 2(4x-1)^2)e^{(4x^2-2x)} \qquad \text{(1 mark)} \end{aligned}$$

(b) Use the method of substitution: Letting $u = 4x^2 - 2x$, then $du = (8x-2)dx$ (1 mark). Now

$$\begin{aligned} \int (4x-1)e^{(4x^2-2x)}\, dx &= \int \frac{1}{2}e^u\, du \\ &= \frac{1}{2}e^u + c \\ &= \frac{1}{2}e^{(4x^2-2x)} + c. \qquad \text{(1 mark)} \end{aligned}$$

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26. Solve the following inequality. Write your solution in interval notation.

$$\frac{2y-1}{y-1} < 4.$$

(3 marks)

Solution

First we note $y \neq 1$.

Consider $y < 1$. Then $y - 1 < 0$ so that

$$\begin{aligned} 2y - 1 &> 4(y-1) \\ 3 &> 2y \\ \frac{3}{2} &> y \end{aligned}$$

giving $y \in (-\infty, 1)$. (1 mark)

For $y > 1$ we get $y - 1 > 0$ such that

$$\begin{aligned} 2y - 1 &< 4(y-1) \\ 3 &< 2y \\ \frac{3}{2} &< y \end{aligned}$$

so $y \in (3/2, \infty)$. (1 mark)

The complete solution is $y \in (-\infty, 1) \cup (3/2, \infty)$ (1 mark).

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27. Let $f(x) = \dfrac{1}{\sqrt{2x-1}}$ and let $g(x) = x^2$.

(a) State the domain and range of $f$. (1 mark)

(b) State the domain and range of $g$. (1 mark)

(c) Determine $(g \circ f)(x)$ and state its domain and range. (2 marks)

(d) Is $g$ a one-to-one function? Explain your answer. (1 mark)

Solution

(a) The domain of $f$ is $(1/2, \infty)$ and the range is $(0, \infty)$. (1 mark)

(b) The domain of $g$ is $\mathbb{R}$ and the range is $[0, \infty)$. (1 mark)

(c) $(g \circ f)(x) = g(f(x)) = g(1/\sqrt{2x-1}) = \dfrac{1}{2x-1}$. (1 mark)
The domain of $(g \circ f)(x)$ includes those values of $x$ in the domain of $f$ such that $1/\sqrt{2x-1}$ is in the domain of $g$. Since the domain of $g$ is $\mathbb{R}$ (or $(-\infty, \infty)$), then the domain of $(g \circ f)(x)$ is the domain of $f$, that is $(1/2, \infty)$.
(0.5 mark)
The range of $(g \circ f)(x)$ is $(0, \infty)$. (0.5 mark)

(d) $g(x)$ is not $1-1$ as $g(x) = g(-x)$. (1 mark)

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28. Let $f(x) = \dfrac{x+1}{x-1}$.

(a) Use the definition of the derivative as a limit to determine the slope of the tangent line at $x = 0$. (3 marks)

(b) Find the equation of the tangent line to $f$ at $x = 0$. (2 marks)

Solution

(a) Using the definition from the Reference Page we have

$$\begin{aligned}
f'(0) &= \lim_{h\to 0} \frac{f(h) - f(0)}{h} \\
&= \lim_{h\to 0} \frac{\frac{h+1}{h-1} - \frac{1}{-1}}{h} \qquad \text{(1 mark)} \\
&= \lim_{h\to 0} \frac{1}{h}.\left(\frac{h+1}{h-1} + 1\right) \\
&= \lim_{h\to 0} \frac{1}{h}.\left(\frac{h+1}{h-1} + \frac{h-1}{h-1}\right) \\
&= \lim_{h\to 0} \frac{1}{h}.\frac{2h}{h-1} \qquad \text{(1 mark)} \\
&= \lim_{h\to 0} \frac{2}{h-1} \\
&= -2 \qquad \text{(1 mark)}
\end{aligned}$$

(b) The slope of the tangent line at $x = 0$ is $-2$, and $f(0) = -1$. (1 mark)
Hence the equation of the tangent line to $f$ at $x = 0$ is given by

$$\begin{aligned}
-2 &= \frac{y+1}{x-0} \\
y &= -2x - 1. \qquad \text{(1 mark)}
\end{aligned}$$

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29. Let $f(x) = 2x^2 + 3x$.

(a) Evaluate the Riemann sum $A_L = \sum_{i=1}^{n} f(x_{i-1})\Delta x$ for $f(x)$, with $1 \leq x \leq 2$, having 10 subintervals. (3 marks)

(b) Evaluate the definite integral $\int_0^1 f(x)\ dx$. (1 mark)

Solution

(a)

$$\begin{aligned} \Delta x &= \frac{2-1}{10} = \frac{1}{10}. \qquad \text{(0.5 marks)} \\ x_i &= 1 + \frac{i}{10} \\ &= \frac{i+10}{10}. \qquad \text{(0.5 marks)} \end{aligned}$$

Then

$$\begin{aligned} f(x_{i-1}) &= 2x_{i-1}^2 + 3x_{i-1} \\ &= 2\frac{(i+9)^2}{100} + 3\frac{i+9}{10} \\ &= \frac{i^2+18i+81}{50} + \frac{15i+135}{50} \\ &= \frac{i^2+33i+216}{50}. \qquad \text{(1 mark)} \end{aligned}$$

Using the summation formulae given on the reference page,

$$\begin{aligned} A_R &= \sum_{i=1}^{10} \left( \frac{i^2}{50} + \frac{33i}{50} + \frac{216}{50} \right) \frac{1}{10} \\ &= \frac{1}{500} \sum_{i=1}^{10} i^2 + \frac{33}{500} \sum_{i=1}^{10} i + \frac{216}{50} \qquad \text{(0.5 marks)} \\ &= \frac{1}{500} . \frac{10 \times 11 \times 21}{6} + \frac{33}{500} . \frac{10 \times 11}{2} + \frac{216}{50} \\ &= \frac{77}{100} + \frac{363}{100} + \frac{432}{100} \\ &= 8.72. \qquad \text{(0.5 marks)} \end{aligned}$$

(b)

$$\begin{aligned}
\int_0^1 (2x^2 + 3x)\, dx &= \left[\frac{2}{3}x^3 + \frac{3}{2}x^2\right]_0^1 \qquad (0.5 \text{ mark}) \\
&= \left(\frac{2}{3} + \frac{3}{2}\right) \\
&= \frac{13}{6} \\
&= 2\frac{1}{6} \qquad (0.5 \text{ mark})
\end{aligned}$$

30. (a) Let $z = 1 + i$. Use De Moivre's Theorem to evaluate $z^4$. (2 marks)

    (b) One solution of $z^3 - 3z^2 + 4z - 2 = 0$ is $z = 1 + i$. Find all the other solutions of this equation over $\mathbb{C}$. (3 marks)

Solution

(a) $|z| = \sqrt{2}$, $\text{Arg}(z) = \dfrac{\pi}{4}$.

In polar form $z = \sqrt{2}\ \text{cis}\left(\dfrac{\pi}{4}\right)$. (1 mark)

Now $z^4 = \sqrt{2}^4\ \text{cis}\left(\dfrac{4\pi}{4}\right) = 4\ \text{cis}\,(\pi) = -4$. (1 mark)

(b) As $z = 1+i$ is a solution then a second solution is $z = 1-i$ since the polynomial has real co-efficients (0.5 marks). Next write

$$\begin{aligned} (az+b)(z-(1+i))(z-(1-i)) &= 0. \qquad \text{(0.5 marks)} \\ (az+b)(z^2-2z+2) &= 0 \\ az^3+(b-2a)z^2+(2a-2b)z+2b &= 0 \end{aligned}$$

Comparing with the original polynomial, we see that $a = 1$, $b = -1$, (1 mark) so

$$z^3 - 3z^2 + 4z - 2 = (z-1)(z-(1+i))(z-(1-i)).$$

The solutions are $z = 1,\ 1+i,\ 1-i$. (1 mark)

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REFERENCE PAGE

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\det(A) = ad - bc$.

If $\det(A) \neq 0$, then $A^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

If $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $||\,\mathbf{v}\,|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ and $\hat{\mathbf{v}} = \dfrac{1}{||\,\mathbf{v}\,||}\mathbf{v}$.

Scalar product: $\mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3$ .

Vector product: $\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

Cosine Law: $a^2 = b^2 + c^2 - 2bc\cos A$ Sine Law: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Series: $\displaystyle\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$, $\displaystyle\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$.

If $\{a_j\}_{j=1}^{\infty}$ is a geometric sequence with first term $a$ and common ration $r$, then

$$S_n = \sum_{j=1}^{n} a_j = \frac{a(r^n - 1)}{r - 1}.$$

The definition of the derivative $f'(a)$ of a function $f$ at the number $a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Differentiation rules:

$$(fg)' = f'g + fg', \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \quad (f \circ g)'(x) = f'(g(x)).g'(x).$$

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Trigonometric identities:

$$\begin{aligned} \sin(\theta + \phi) &= \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi) \\ \cos(\theta + \phi) &= \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi) \end{aligned}$$

Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$.

De Moivre's theorem: If $z = r(\cos(\theta) + i\sin(\theta))$ then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$.

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