11. Given matrices $A = \begin{pmatrix} 1 & 4 & 0 \\ -2 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 3 & -2 \\ 1 & 3 & 0 \end{pmatrix}$, evaluate the following where possible:

    (a) $A + B$ (1 marks)

    (b) $A + B^T$ (1 mark)

    (c) AB (1 mark)

    (d) $A^T B$ (2 marks)

COPYRIGHT RESERVED

12. Find the values of x and y such that $\det(A) = 0$ where

$$A = \begin{pmatrix} 1 & 0 & -2 \\ -2 & 3 & x \\ y & 0 & 2 \end{pmatrix}.$$

(2 marks)

COPYRIGHT RESERVED

13. A triangle has sides of length 3,7,9. Determine the three angles of the triangle, measured in degrees. (Give answers to an accuracy of two decimal places) (3 marks)

COPYRIGHT RESERVED

14. Let $\mathbf{u}$ be the vector which has magnitude 8 and direction $\frac{5\pi}{4}$. Let $\mathbf{v} = 3\mathbf{i} - 3\sqrt{3}\mathbf{j}$. Evaluate $\mathbf{u}.\mathbf{v}$. (Give answer to an accuracy of two decimal places.) (2 marks)

COPYRIGHT RESERVED

15. Draw a parallelogram with vertices A, B, C, D such that $\overrightarrow{AD} = \overrightarrow{BC}$. Draw a line from B to D and a line from A to C, and let E denote the point of intersection of these two lines. Show that E is the midpoint of both lines by proving that

$$\overrightarrow{BE} = \frac{1}{2}\overrightarrow{BD}, \qquad \overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}.$$

(4 marks)

COPYRIGHT RESERVED

16. Two lengths of rope, one 30 cm long and the other 40 cm long, are attached to a ceiling a distance of 50 cm apart. The other ends of the rope are joined together and attached to a basket weighing 0.5 kg, which hangs freely. Determine the tension in each rope. (The acceleration due to gravity should be taken as $g = 9.8$ m/s^2.) (5 marks)

COPYRIGHT RESERVED

17. On a snooker table (assumed to be frictionless), the white ball is travelling north at a speed of 1 m/s and the black ball is at rest. They collide, and after the collision the white ball rolls in the north-west direction while the black ball moves in the north-east direction at 0.6 m/s. If the mass of the white ball is 10 grams, what is the mass of the black ball in grams (to 2 decimal places). (5 marks)

COPYRIGHT RESERVED

18. This question is about proving the Sine Law and the Cosine Law, with reference to the figure below.

(a) Find two expressions for the length d, one in terms of the angle A and one in terms of the angle B. (2 marks)

(b) From the results of (a), deduce that the Sine Law

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

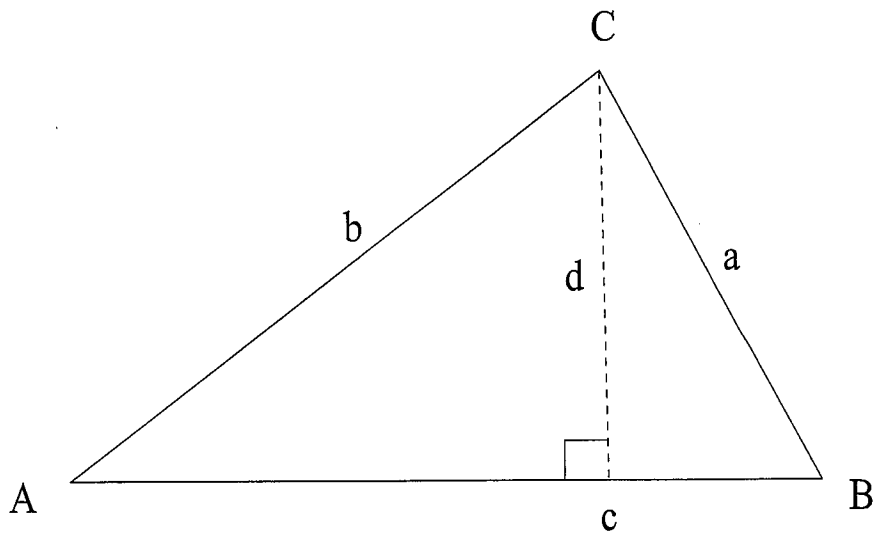
holds. (1 mark)

(c) Choosing the point A to be the origin, the co-ordinates of the point C are $(b\cos A, b\sin A)$, while the co-ordinates of the point B are $(c, 0)$. In terms of these cordinates, calculated the magnitude of $\overrightarrow{BC}$. (3 marks)

(d) From the results of (c), deduce that the Cosine Law

$$a^2 = b^2 + c^2 - 2bc\cos A$$

holds. (1 mark)



COPYRIGHT RESERVED

18. continued

COPYRIGHT RESERVED

19. Find the exact value of $\sin\left(\frac{17\pi}{12}\right)$ (where the angle is measured in radians). (3 marks)

COPYRIGHT RESERVED

20. Uschi and Hans are two MATH1050 students who decide to celebrate at The Red Room after finishing their midsemester exam. Uschi drinks 5 beers and 1 cocktail costing a total of \$41. Hans drinks 3 beers and 3 cocktails which costs him \$45. Using matrices, determine the price of a beer and the price of a cocktail. (4 marks)

COPYRIGHT RESERVED

END OF EXAMINATION