## DEPARTMENT OF MATHEMATICS

## MATH2000 Assignment 1 Summer 2010-2011

Submit your answers - along with this cover sheet - at the end of your tutorial on Wednesday, December 8, 2010.

Note that you may find some of these problems challenging. Attendance at weekly tutorials is assumed.

Family name:

Given names:

Student number:

Marker's use only

Each question marked out of 3.

- Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
- Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique. This topic needs more attention!
- Mark of 2: You have the right approach, but need to fine tune some aspects of your calculations.
- Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with well-executed calculations. A mathematician in the making?

Q1: Q2: Q3: Q4:

Total (out of 12):

(1) Consider the initial value problem

$$2x + 4y + (4x - 2y)\frac{dy}{dx} = 0, \quad y(x_0) = y_0$$

Using the existence and uniqueness criteria given in lectures, give the region (call it D) in the x-y plane consisting of all points  $(x_0, y_0)$  such that there is a unique solution. Choose a point in D as your initial condition, show that the equation is exact, then use this fact to solve the associated initial value problem.

(2) Solve the initial value problem

$$y'' + 4y' + 4y = e^{-2x} \left(2 + \frac{1}{x^2}\right), \quad y(1) = 0, \ y'(1) = 0.$$

- (3) Express the inverse hyperbolic secant function (usually written as sech<sup>-1</sup> or arcsech) in terms of the natural logarithm. Note that the domain is (0, 1] and the range is [0,∞).
- (4) Evaluate the integral

$$\int_{-2/3}^{-1/3} \frac{dx}{x\sqrt{1-x^2}}.$$

Show all working.