DEPARTMENT OF MATHEMATICS

MATH2000
Assignment 3
Summer 2010-2011

Submit your answers - along with this cover sheet - at the end of your tutorial on Wednesday, January 12, 2011.

Note that you may find some of these problems challenging. Attendance at weekly tutorials is assumed.

Family name:

Given names:

Student number:

Marker's use only

Each question (or part question) marked out of 3.

- Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
- Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique. This topic needs more attention!
- Mark of 2: You have the right approach, but need to fine tune some aspects of your calculations.
- Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with well-executed calculations. A mathematician in the making?

Q1 (a):
Q1 (b):

Q2:

Q3:

Q4:

Q5:

Total (out of 18):

(1) Consider the expression

$$I = \int_0^1 \int_0^{2z} \int_z^1 dx\ dy\ dz + \int_0^1 \int_{2z}^{1+z} \int_{y-z}^1 dx\ dy\ dz.$$

(a) Determine the value of I by evaluating the iterated integrals given.

(b) Rewrite I using the order of integration $dy\ dx\ dz$ and evaluate the new expression, verifying your answer in part (a).

(2) Consider a torus whose equation in terms of spherical coordinates (r, θ, ϕ) is $r = 2\sin\phi$ for $0 \leq \phi \leq \pi$ and $0 \leq \theta \leq 2\pi$. Determine the volume of the region bounded by the torus by calculating a triple integral using spherical coordinates.

(3) Consider set of variables (u, v, w) such that

$$\begin{aligned} x &= (h + u\sin v)\cos w, \\ y &= (h + u\sin v)\sin w, \\ z &= u\cos v, \end{aligned}$$

for some fixed, non-negative h. If we specify $h \geq 1$, then the region represented by the set of points

$$\{(u, v, w) \mid 0 \leq u \leq 1,\ 0 \leq w \leq 2\pi,\ 0 \leq v \leq 2\pi\},$$

describes a torus (doughnut!) of radius 1, whose centre is a distance h from the (x, y, z) origin. Determine the volume of this torus by calculating a triple integral using the (u, v, w) coordinate system.

(4) Consider a solid cylinder of radius R, a cylindrical shell of outer radius R and inner radius a, and a spherical shell with outer radius R and inner radius a. All three objects have the same constant density. Find the heights of the cylinders such that all three objects have the same moments of inertia when rolling down a slope. Which cylinder has the larger height?

(5) Let C be a circle of radius 1 centred at the point $(x, y) = (0, 1)$. Consider the vector field $\boldsymbol{F}(x, y) = x^2\boldsymbol{i} + xy\boldsymbol{j}$. Calculate the two line integrals over C,

$$\oint_C \boldsymbol{F} \cdot \boldsymbol{T}\ ds \text{ and } \oint_C \boldsymbol{F} \cdot \boldsymbol{n}\ ds,$$

where T is the unit tangent vector to C traversed in an anticlockwise direction, and $\boldsymbol{n}$ is a unit normal vector to C directed away from the centre of the circle.