DEPARTMENT OF MATHEMATICS

MATH2000
Assignment 4
Summer 2010-2011

Submit your answers - along with this cover sheet - at the end of your tutorial on Wednesday, February 2, 2011.

Note that you may find some of these problems challenging. Attendance at weekly tutorials is assumed.

Family name:

Given names:

Student number:

Marker's use only

Each question (or part question) marked out of 3.

- Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
- Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique. This topic needs more attention!
- Mark of 2: You have the right approach, but need to fine tune some aspects of your calculations.
- Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with well-executed calculations. A mathematician in the making?

Q1:

Q2:

Q3:

Q4:

Q5:

Total (out of 15):

(1) Evaluate the line integral

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r},$$

where $\boldsymbol{F}(x, y) = (1 + 2xy)\boldsymbol{i} + (1 + x^2)\boldsymbol{j}$ and C is the curve parameterised by

$$\boldsymbol{r}(t) = t\boldsymbol{i} + \cos(\frac{\pi}{2}t^2)\boldsymbol{j}, \quad 0 \le t \le 1.$$

(2) Calculate the net outward flux of the vector field $\boldsymbol{F}(x, y, z) = 3xy^2\boldsymbol{i} + x\cos z\boldsymbol{j} + z^3\boldsymbol{k}$ across the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

(3) Suppose that $f(x, y, z) = g(\sqrt{x^2 + y^2 + z^2})$, where g is a function of one variable such that $g(t) = t - 5$. Evaluate

$$\oiint_S f(x, y, z)\ dS,$$

where S is the sphere $x^2 + y^2 + z^2 = 9$.

(4) Verify that Stokes' theorem is true for the vector field $\boldsymbol{F}(x, y, z) = x\boldsymbol{i} + y\boldsymbol{j} + xyz\boldsymbol{k}$, and the surface S which is part of the plane $2x + y + z = 2$ that lies in the octant $x, y, z \ge 0$, oriented upward (i.e. the unit normal vector to S has a positive $\boldsymbol{k}$ component). Note that

$$\boldsymbol{r}(u, v) = (1 - u)\boldsymbol{i} + 2u(1 - v)\boldsymbol{j} + 2uv\boldsymbol{k}, \quad 0 \le u \le 1,\ 0 \le v \le 1$$

gives a parameterisation of S.

(5) Find a PLU decomposition for the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & -3 \\ 2 & 6 & 2 & 9 \\ 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 4 \end{pmatrix}.$$