DEPARTMENT OF MATHEMATICS

MATH2000
Cylindrical and Spherical Coordinates

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(1) For spherical coordinates

$$\begin{aligned} x =& r\cos\theta\sin\phi \\ y =& r\sin\theta\sin\phi \\ z =& r\cos\phi \end{aligned}$$

find expressions for r, θ and ϕ in terms of x, y and z.

(2) Express the following surfaces in terms of spherical coordinates: (a) $x^2 + y^2 + z^2 = 9$, (b) $z = \sqrt{3x^2 + 3y^2}$, (c) $y = x$, (d) $z = h$ and (e) $x^2 + y^2 = 4$.

(3) Consider a planet of radius $r = a$ which is covered in a layer of ice of height $h = b - a$, so that the outer radius of the ice layer is $r = b$. The density of the ice is assumed to be $\rho(x, y, z) = kz^2$ for $a \leq r \leq b$. Find the mass of ice on the planet.

(4) Find the mass of the solid defined by the region contained within the cylinder $x^2 + y^2 = 1$ below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$. The density at any given point in the region is proportional to the distance from the axis of the cylinder. In lectures (workbook page 99) the integral was performed in the order $\int\int\int dzdrd\theta$. For this problem perform the integration with respect to r first, i.e., $\int\int\int drdzd\theta$.

(5) Find the centre of mass of the solid hemisphere bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and the x-y plane whose density function is

$$\rho(x, y, z) = \rho_0 \left(1 + \frac{1}{a}\sqrt{x^2 + y^2 + z^2}\right).$$

Hint: since the density function is symmetric about the z-axis inside the hemisphere, you can assume the centre of mass lies somewhere on the z-axis (ie, the x and y coordinates are both 0).

(6) (Stewart ed. 6, p1040, Q23)

(a) Use cylindrical coordinates to find the volume of the region E bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

(b) Find the centroid of E (the centre of mass in the case where the density is constant).