DEPARTMENT OF MATHEMATICS

MATH2000
Double Integrals in Rectangular Coordinates (x-y coordinates)

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(1) Evaluate the following integrals

(a) $\displaystyle\int_0^2 \int_0^1 (x+y)\ dx\ dy$

(b) $\displaystyle\int_0^1 \int_0^2 (x^4y^5+y)\ dx\ dy$

(c) $\displaystyle\int_0^1 \int_0^{2x} 2y^2\ dy\ dx$

(d) $\displaystyle\int_0^1 \int_y^{\sqrt{y}} 2x\ dx\ dy$

(2) Find the volume of the solid bounded by the planes $x-y-z=0$, $y=0$, $z=0$ and $x=4$. Note that this problem is equivalent to finding the volume below the surface $z=f(x,y)=x-y$ and above the region in the x-y plane bounded by the lines $y=0$, $y=x$ and $x=4$.

(3) Calculate $\displaystyle\iint_D x\ dA$ where D is the region in the x-y plane bounded by $y=2-x$ and $y=x^2$.

(4) Evaluate the integral $\displaystyle\int_0^1 \int_y^1 \cos(x^2)\ dx\ dy$.

(5) Find the volume under the surface $z=e^{-x^2}$ and above the triangle in the x-y plane with vertices $(0,0)$, $(2,0)$ and $(2,2)$.

(6) Find the net volume under the surface $z=x+y$ and above the region in the x-y plane lying between the two parabolas $y=x^2$ and $y=2-x^2$. In otherwords, calculate

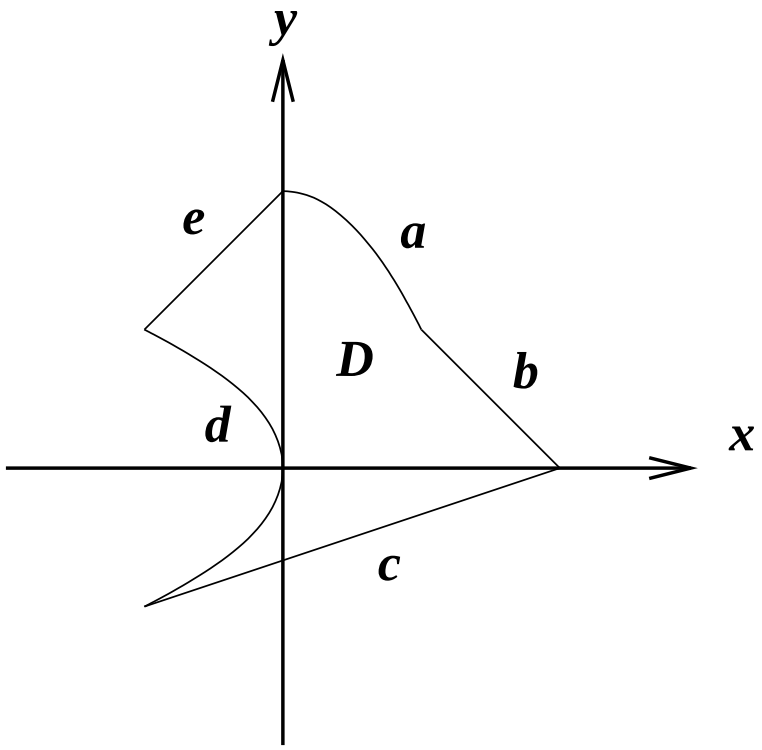
$$\iint_D (x+y)\ dA,$$

where D is the specified region in the x-y plane.

(7) By changing the order of integration find an equivalent expression for $\displaystyle\int_0^1 \left(\int_x^1 f(x,y)dy\right) dx$.

(8) By changing the order of integration find an equivalent expression for $\displaystyle\int_0^1 \left(\int_{1-x}^1 f(x,y)dy\right) dx$.

(9)



The region D in the x-y plane is bounded by five curves as labelled in the diagram above. These curves are as follows:

$$\begin{aligned} a: \quad y &= -x^2 + 2 \\ b: \quad y &= -x + 2 \\ c: \quad x &= 3y + 2 \\ d: \quad x &= -y^2 \\ e: \quad y &= x + 2. \end{aligned}$$

Express D as a union of regions of type I or type II and show how you would evaluate the double integral

$$\iint_D f(x,y)dA$$

in terms of iterated integrals.