DEPARTMENT OF MATHEMATICS

MATH2000 First Order ODEs

(1) Find the general solution to the equation

$$y' + xy = xy^{-1}$$

using two approaches:

- (a) The ODE is separable.
- (b) Multiply both sides of the ODE by ye^{x^2} to make it exact.
- (2) Show that the equation

$$2x^2 + xy^2 + x^2yy' = 0$$

is exact then use this fact to solve the initial value problem with y(1) = -2. Note you should be able to write y(x) explicitly.

(3) Find the general solution to the following ODE:

$$(x^2 - 2x)y' = 2(x - 1)y.$$

Now consider the initial value problem with $y(x_0) = y_0$ and determine all initial conditions (x_0, y_0) such that the initial value problem has (a) no solutions, (b) more than one solution, and (c) precisely one solution. What do the existence and uniqueness criteria from lectures tell us about the solutions?

(4) Let a, b, c, d be constants. Under what condition is the equation

$$ax + by + (cx + dy)\frac{dy}{dx} = 0$$

exact? Impose this condition and solve the exact equation.