DEPARTMENT OF MATHEMATICS

MATH2000
Flux integrals and Gauss' divergence theorem

---

(1) Verify Gauss' theorem for the vector field $\boldsymbol{F} = x^3\boldsymbol{i} + y^3\boldsymbol{j} + z^3\boldsymbol{k}$ across the solid (upper) hemisphere of radius 1.

(2) Calculate the net outward flux of the vector field $\boldsymbol{F} = \dfrac{1}{x}\boldsymbol{i}$ across the surface of the unit sphere centred at the origin.

(3) Verify Gauss' divergence theorem by calculating the outward flux of the vector field $\boldsymbol{F} = x\boldsymbol{i} + 12y\boldsymbol{j} + 3z\boldsymbol{k}$ through the surface of the box given by

$$B = \{(x, y, z) \mid 1 \le x \le 3,\ 0 \le y \le 1,\ 3 \le z \le 5\}.$$

(4) Find the flux of the vector field $\boldsymbol{F} = 3x\boldsymbol{i} + 4y\boldsymbol{j} + 5z\boldsymbol{k}$ over the surface S of a sphere of radius 5 centred at the origin.

(5) Find the flux of $\boldsymbol{F} = x\boldsymbol{i} + 3y\boldsymbol{j} + 6z\boldsymbol{k}$ across the surface of the solid above the x-y plane (ie. $z \ge 0$) bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 2$.

(6) Find the flux of $\boldsymbol{F} = (x^3 + xy^2 + xz^2)\boldsymbol{i} + (x^2y + y^3 + yz^2)\boldsymbol{j} + (x^2z + y^2z + z^3)\boldsymbol{k}$ across the surface of the sphere of radius a centred at the origin.