DEPARTMENT OF MATHEMATICS

MATH2000
Gaussian elimination, LU and PLU decomposition

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(1) Find a matrix in row echelon form equivalent to:

(a) $\begin{pmatrix} 0 & 0 & 0 \\ 2 & 4 & 0 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ **(c)** $\begin{pmatrix} 0 & 1 & 3 \\ 2 & 0 & 0 \\ 4 & 0 & 0 \end{pmatrix}$ **(d)** $\begin{pmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{pmatrix}$

(2) Find a matrix in row echelon form equivalent to the complex matrix $\begin{pmatrix} 2+i & -1+2i & 2 \\ 1+i & -1+i & 1 \\ 1+2i & -2+i & 1+i \end{pmatrix}$

(3) Use the Gauss method to find A^{-1} if $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$.

(4) Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix}$.

Use the LU decomposition to solve $A\mathbf{x} = \mathbf{b}$.

(5) In this question, all matrices are square $n \times n$ ones.

- **(a)** Show that the product of two upper triangular matrices (that is, two matrices with 0 everywhere below the main diagonal) is upper triangular, and show that the inverse of an upper triangular matrix is upper triangular.
- **(b)** Repeat part **(a)**, but for lower triangular matrices. (So take transposes and use part **(a)**.)
- **(c)** If L is lower triangular with 1 everywhere on the main diagonal, show that L^{-1} also has 1 everywhere on its main diagonal.

(6) Let

$$A = \begin{pmatrix} 3 & 1 & 0 & -5 \\ -6 & -1 & -1 & 10 \\ 3 & 3 & 2a-2 & a-5 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

where a is a non-zero constant.

- **(a)** Find an LU decomposition for the matrix A.
- **(b)** Show that A has zero determinant.

(c) Use part (a) to solve the system

$$A\boldsymbol{x} = \boldsymbol{b}$$

where

$$\boldsymbol{b} = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

(d) Use part (a) again to solve the system

$$A\boldsymbol{x} = \boldsymbol{b}$$

where

$$\boldsymbol{b} = \begin{pmatrix} 2 \\ -5 \\ 0 \\ 1 \end{pmatrix}$$

(7) Find a PLU decomposition for the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 8 & 12 & 17 \\ 3 & 6 & 12 & 14 \\ 2 & 9 & 11 & 12 \end{pmatrix}$$

and hence calculate its determinant.

(8) Let

$$A = \begin{pmatrix} -1 & -3 & -4 \\ 3 & 10 & -10 \\ -2 & -4 & a \end{pmatrix}$$

where $a \in \mathbb{R}$ is a constant.

(a) Find an LU decomposition for the matrix A.

(b) For what value of a does the matrix A have zero determinant?

(c) Set $a = 11$ then use your answer in part (a) to solve the system $A\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{b} = \begin{pmatrix} -6 \\ -3 \\ 9 \end{pmatrix}$.

(9) Find a PLU decomposition for the matrix $\begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$.