DEPARTMENT OF MATHEMATICS

MATH2000
Green's theorem, introduction to flux.

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(1) Determine the work done by a force $\boldsymbol{F} = (y - xy)\boldsymbol{i} + x^2\boldsymbol{j}$ when a particle moves counter clockwise around a rectangle with vertices $(0,0)$, $(2,0)$, $(2,1)$ and $(0,1)$. The problem is best solved using *Green's Theorem in the plane.*

(2) (Stewart, p1096 Q6)

Use Green's theorem to evaluate the line integral

$$\int_C x^2y^2\ dx + 4xy^3\ dy,$$

counterclockwise along the closed curve C, where C is the triangle with vertices $(0,0)$, $(1,3)$ and $(0,3)$.

(3) (Stewart, p1096 Q8)

Use Green's theorem to evaluate the line integral

$$\int_C xe^{-2x}\ dx + (x^4 + 2x^2y^2)\ dy,$$

where C is the positively oriented curve defined by the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(4) Calculate the net outward flux of $\boldsymbol{v} = -y\boldsymbol{i} + x\boldsymbol{j}$ across the boundary of the rectangle $\{(x,y) \mid 2 \le x \le 4,\ 2 \le y \le 6\}$.

(5) Calculate the net outward flux of $\boldsymbol{v} = (x+y)\boldsymbol{i} - (x^2+y^2)\boldsymbol{j}$ across the boundary of the triangle with vertices $(1,0)$, $(0,1)$ and $(-1,0)$.

(6) Let $\alpha > 0$ be a constant with dimensions s^{-1}. Calculate the net outward flux (in two dimensions) of the velocity field $\boldsymbol{v} = \alpha x\boldsymbol{i}$ across the boundary of the rectangle $\{(x,y) \mid 1 \le x \le 3,\ 2 \le y \le 5\ \}$. What are the dimensions of the flux in this case?

(7) (Stewart, p1096 Q3) Evaluate the line integral $\oint_C xy\ dx + x^2y^3\ dy$, where C is anticlockwise around the triangle with vertices $(0,0)$, $(1,0)$ and $(1,2)$.