DEPARTMENT OF MATHEMATICS

MATH2000 Hyperbolic Functions

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(1) Find $\displaystyle\int \frac{x}{\sqrt{1+x^4}}dx$. Hint: you may need more than one substitution.

(2) Show that $\operatorname{arcosh}(x) = \ln(x + \sqrt{x^2-1})$. Note that the domain of $\operatorname{arcosh}(x)$ is $[1, \infty)$ and the range is $[0, \infty)$.

(3) Find $\displaystyle\int \operatorname{arsinh}(x)\, dx$. Hint: use integration by parts.

(4) Find the general solution to the differential equation

$$\frac{d^2y}{dx^2} = 1 - \left(\frac{dy}{dx}\right)^2, \quad \left|\frac{dy}{dx}\right| < 1.$$

(5) Consider the ODE

$$\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^2 + 4 = 0.$$

Find the general solution satisfying the condition $\left|\dfrac{dy}{dx}\right| > 2$ for all $x \neq 0$.

(6) (a) Show $e^{ix} = \cos x + i \sin x$ using Taylor series.

(b) Show that

$$\sinh 2x = 2 \sinh x \cosh x$$

(c) Use the results of parts (a) and (b) to show that

$$\sin 2x = 2 \sin x \cos x.$$