## DEPARTMENT OF MATHEMATICS

## MATH2000 Quadratic forms

(1) Write the following quadratic equations in the matrix form

$$\underline{x}^T A \underline{x} + K \underline{x} + c = 0,$$

where A is a symmetric  $2 \times 2$  matrix,  $\underline{x}$  is the coordinate vector  $\begin{pmatrix} x \\ y \end{pmatrix}$ , K is a  $1 \times 2$  matrix, and c is a real number.

- (a)  $2x^2 3xy + 4y^2 7x + 2y + 7 = 0$ , (b)  $x^2 - xy + 5x + 8y - 3 = 0$ , (c) 5xy = 8, (d)  $4x^2 - 2y^2 = 7$ , (e)  $y^2 + 7x - 8y - 5 = 0$ .
- (2) Express the quadratic form  $Q(x,y) = 5x^2 + 2y^2 + 4xy$  as a linear combination of two squares.
- (3) A quadratic form  $\underline{x}^T A \underline{x}$  is called positive definite if  $\underline{x}^T A \underline{x} > 0$  for all  $\underline{x} \neq 0$ , and a symmetric matrix A is called a positive definite matrix if the associated quadratic form is positive definite. Show that the quadratic form  $f(x, y) = 5x^2 2xy + 5y^2$  is positive definite.
- (4) Consider the conic section

$$2x^2 - 4xy - y^2 - 4x - 8y = -14.$$

Rotate and translate the coordinate axes to write it in standard form. Hence name the type of conic section.

(5) Consider the conic section

$$9x^2 - 4xy + 6y^2 - 10x - 20y = 5.$$

Rotate and translate the coordinate axes to write it in standard form. Hence name the type of conic section.