DEPARTMENT OF MATHEMATICS

MATH2000
Nonhomogeneous Linear Second Order ODEs

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(1) Find the general solution of $y'' + y' - 2y = 10\cos x$.

(2) Find the general solution of $y'' - 3y' + 2y = \exp(2x)$ with $y(0) = 2$ and $y'(0) = 0$.

(3) Find the general solution of $y'' + 2y' = 1$.

(4) Solve the initial value problem

$$y'' - y' - 6y = 1 + 6x^2, \quad y(0) = \frac{1}{3},\ y'(0) = \frac{4}{3}.$$

(5) Solve the initial value problem

$$y'' + y = \sin x, \quad y(0) = 1,\ y'(0) = 0.$$

(6) In an LRC circuit the voltage drop across an inductor, capacitor and resistor is given by

$$V_I = L\frac{di}{dt},\ V_C = \frac{1}{C}\int_0^t i(s)ds \text{ and } V_R = iR.$$

If they are arranged in series Kirchoff's law states that

$$V_I + V_C + V_R = V \text{ or } L\frac{di}{dt} + \frac{1}{C}\int^t i(s)ds + V_R = iR = V(t)$$

where $V(t)$ is the applied voltage. Taking the derivative gives

$$L\ddot{i} + R\dot{i} + \frac{1}{C}i = \frac{dV}{dt}.$$

Suppose $L = 1$, $R = 4$ and $C = \frac{1}{8}$.

(a) Find a general solution if the voltage is constant $\frac{dV}{dt} = 0$

(b) Find a general solution if the voltage is such that $\frac{dV}{dt} = 1$

(c) Find a general solution if the voltage is such that $\frac{dV}{dt} = 20\cos 2t$

In each case find the solution as $t \to \infty$, this is called the steady state solution.

(7) Solve the initial value problem

$$y'' + 4y = \cos(2x), \quad y(0) = 2, \quad y'(0) = 2.$$

(8) Solve the initial value problem

$$y'' - 6y' + 9y = 4e^{3x} + 14\cos x, \quad y(0) = 4, \quad y'(0) = 5.$$

(9) Find the general solution to the nonhomogeneous equation

$$x^2y'' + xy' - n^2y = x^m, \quad x > 0,$$

where m and $n \neq 0$ are any real numbers such that $m^2 \neq n^2$.

Hint: to find the general solution to the corresponding homogeneous equation (y_H), assume the solution is of the form $y = x^\lambda$. You should end up with a characteristic equation which you can solve for λ, after which you should be able to write down y_H. Then use variation of parameters to find y_P. Don't forget to rewrite the equation in standard form (ie. with 1 as the coefficient of y'').

(10) Find the general solution to

$$y'' - 2y' + y = e^x/x^3,$$

(11) Find the general solution of $y'' - 2y' + y = x^{\frac{3}{2}}e^x$.

(12) Find the general solution of $y'' + y = \cot x$, $0 < x < \pi/2$.