MATH2100 Assignment 7
(= MATH2011 Assignment 2)

1.* In a spherical balloon of constant radius R, the density $\rho(t) = \rho_0 e^{-\alpha t}$ of Helium decreases steadily with time t as Helium leaks out through the surface and is replaced by air. Here ρ_0 and α are positive constants. What is the total mass $Q(t)$ of Helium inside the balloon at time t?

Assuming that the Helium mass flux vector inside the balloon has the general form $\vec{J}(\vec{r}, t) = f(r, t)\vec{r}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, and assuming that ρ and $\vec{J}$ satisfy the conservation equation

$$\partial\rho/\partial t + \mathrm{div}\vec{J} = 0\,,$$

deduce that

$$\vec{J} = \tfrac{1}{3}\alpha\rho_0\, e^{-\alpha t}\, \vec{r}\,.$$

(You may assume also that $\vec{J} = \vec{0}$ at the balloon's centre $\vec{r} = \vec{0}$.)

Use $\vec{J}$ to calculate the flux of Helium mass out over the surface of the balloon per unit time, and check that this has the same value as $-dQ(t)/dt$.

Finally, find a $\Phi(\vec{r}, t)$ such that $\vec{J} = \mathrm{grad}\,\Phi$ and hence or otherwise deduce that $\mathrm{curl}\,\vec{J} = \vec{0}$.

Practice Problems:

Problem Set K9.7 p.409 Numbers 16, 18

Problem Set K9.8 p.413 Numbers 4, 8

Problem Set K9.9 p.416 Numbers 2, 6

Solution to the starred problem #1 to be handed in by 5pm on Monday, October 1 in appropriate box on Level 3, Mathematics Bldg 67.

NB: Use a cover sheet! (Download from www.maths.uq.edu.au/courses/MATH2100 /Tutorials/cover_sheet.pdf)