MATH2100 Assignment 8 (= MATH2011 Assignment 3)

A[★]: Problem Set K12.3 p.547 Number 4 (Use Fourier's method - no need for graphs.)

B^{*}: Let $\eta(x,t) = x^2/(4c^2t)$. Check that

$$\begin{split} \eta_t(x,t) &= -\eta(x,t)/t \,, \quad [\eta_x(x,t)]^2 = \eta(x,t)/(c^2 t) \,, \\ \eta_{xx}(x,t) &= 1/(2c^2 t) \,, \end{split}$$

where η_t denotes $\partial \eta / \partial t$ etc. Hence show that $u(x,t) = t^{-1/2} f(\eta(x,t))$ is a solution of the 1-dimensional heat equation $u_t(x,t) = c^2 u_{xx}(x,t)$ provided that

$$\eta f''(\eta) + (\eta + \frac{1}{2})f'(\eta) + \frac{1}{2}f(\eta) = 0.$$

Check that $f(\eta) = e^{-\eta}$ satisfies this last equation, and hence deduce that

$$u(x,t) = t^{-1/2} e^{-\eta(x,t)} = t^{-1/2} e^{-x^2/(4c^2t)}$$

is a solution of the 1-dimensional heat equation.

Practice problems: Problem Set K12.3 p.546 Numbers 2, 10.

Solution to the two starred problems to be handed in by 5pm on Monday, October 8 in appropriate box on Level 3, Mathematics Bldg 67.

NB: Use a cover sheet! (Download from www.maths.uq.edu.au/courses/MATH2100 /Tutorials/cover_sheet.pdf) Page 2/2