MATH2100 Problem Set 9

(= MATH2011 Problem Set 4)

1: Use Fourier's Method to obtain the solution

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-(\frac{n\pi c}{L})^2 t} \sin(\frac{n\pi x}{L}), \qquad B_n = \frac{2}{L}\int_0^L \sin(\frac{n\pi x}{L} f(x)\, dx$$

of the 1-D Heat Equation for $t > 0$ on the region $0 < x < L$, with the BCs $u(0,t) = 0 = u(L,t)$ and IC $u(x,0) = f(x)$.
Find the first few terms in the series for $u(x,t)$ in the case that $f(x) = \frac{u_0}{L^2} x(L-x)$ (with u_0 const.) for $0 < x < L$.

2: A semi-infinite cylinder of metal lies along the positive x-axis, with its sides and the face at $x = 0$ insulated. The initial temperature distribution is given by $u(x,0) = u_1$ (const.) for $0 < x < L$, and $u(x,0) = u_2$ (const.) for $x > L$. Show that the temperature in the cylinder is given for $t > 0$ by

$$u(x,t) = u_2 + \frac{1}{2}(u_1 - u_2)\left\{\operatorname{erf}\left(\frac{x+L}{\sqrt{4c^2 t}}\right) - \operatorname{erf}\left(\frac{x-L}{\sqrt{4c^2 t}}\right)\right\}.$$

Check directly from this solution that $u_x(0,t) = 0$ for all $t > 0$.

Solution to the two starred problems to be handed in by 5pm on Monday, October 15 in appropriate box on Level 3, Mathematics Bldg 67.

NB: Use a cover sheet! (Download from www.maths.uq.edu.au/courses/MATH2100 /Tutorials/cover_sheet.pdf)