4.1

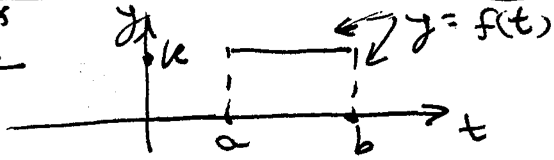
MATH 2010/2100 Solutions 4 AJB

K 6.1 #8* $f(t) = \sin(3t - \tfrac{1}{2}) = \sin(3t)\cos(-\tfrac{1}{2}) + \cos(3t)\sin(-\tfrac{1}{2})$

$= \cos(\tfrac{1}{2})\sin(3t) - \sin(\tfrac{1}{2})\cos(3t)$.

$$\mathcal{L}(f(t)) = \cos(\tfrac{1}{2})\left[\frac{3}{s^2+9}\right] - \sin\tfrac{1}{2}\left[\frac{s}{s^2+9}\right] = \frac{[\cos(\frac{1}{2}) - s.\sin(\frac{1}{2})]}{s^2+9}.$$

K6.1#14*



$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt = \int_a^b k e^{-st}\,dt$$

$$= \frac{k}{-s}\left[e^{-st}\right]_{t=a}^{t=b} = \frac{k}{s}\left[e^{-as} - e^{-bs}\right]$$

K6.1#30* $F(s) = \dfrac{2s+16}{(s-4)(s+4)} = \dfrac{A}{s-4} + \dfrac{B}{s+4}$

$\Rightarrow A(s+4) + B(s-4) = (s^2-16)F(s) = 2s+16$

$s = 4 \Rightarrow 8A = 8+16 = 24 \Rightarrow A = 3$

$s = -4 \Rightarrow -8B = -8+16 = 8 \Rightarrow B = -1$

So $F(s) = \dfrac{3}{s-4} - \dfrac{1}{s+4}$

$\Rightarrow f(t) = \mathcal{L}^{-1}(F(s)) = 3e^{4t} - e^{-4t}$

K6.1#34* $F(s) = \dfrac{2s-56}{(s-6)(s+2)} = \dfrac{A}{s-6} + \dfrac{B}{s+2}$

$\Rightarrow A(s+2) + B(s-6) = 2s - 56$

$s = -2 \Rightarrow -8B = -4-56 = -60 \Rightarrow B = 7\tfrac{1}{2}$.

$s = 6 \Rightarrow 8A = 12 - 56 = -44 \Rightarrow A = -\tfrac{11}{2}$.

So $F(s) = \dfrac{-11/2}{s-6} + \dfrac{15/2}{s+2}$

$\Rightarrow f(t) = \mathcal{L}^{-1}(F(s)) = -\tfrac{11}{2}e^{6t} + \tfrac{15}{2}e^{-2t}$

[Note that this can be regarded as an application of the First Shifting Theorem, as $\mathcal{L}^{-1}\left(\frac{-11/2}{s}\right) = -\frac{11}{2} \Rightarrow \mathcal{L}^{-1}\left(\frac{-11/2}{s-6}\right) = -\frac{11}{2}e^{6t}$ etc.

K6.1#46* $\mathcal{L}(t^n) = \dfrac{n!}{s^{n+1}}$ $n = 0, 1, 2, \dots$

$\mathcal{L}(e^{-t}t^n) = \dfrac{n!}{(s+1)^{n+1}}$ by First Shifting Theorem

So

$$\Rightarrow \mathcal{L}[e^{-t}(a_0 + a_1 t + \dots + a_n t^n)] = \frac{a_0}{s+1} + \frac{1!\,a_1}{(s+1)^2} + \dots + \frac{n!\,a_n}{(s+1)^{n+1}}$$