1. Find two independent vectors for the solution space of the system of equations:

\[ \begin{bmatrix} 2 & 0 & 1 & 0 \\ 3 & 6 & 9 & 5 \\ 1 & 4 & 7 & 2 \\ 0 & 6 & 9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

2. Show that the only solutions of \( \mathbf{A} \mathbf{x} = \mathbf{0} \) are \( \mathbf{x} = \mathbf{0} \). (If \( \mathbf{A} \mathbf{x} = \mathbf{0} \) has only the zero solution, then \( \mathbf{A} \mathbf{x} = \mathbf{b} \) has a solution if and only if \( \mathbf{b} \) is in the column space of \( \mathbf{A} \).

3. Find the nullspace of \( \mathbf{A} \). Is \( \mathbf{A} \) onto?
(c) Find the rank of the matrix $A$ and its transpose $A^T$.

There is no such thing as rank for a transposed matrix.

(d) Find the rank of the matrix $A$ and its inverse $A^{-1}$.

The rank of $A$ is $n - 0$, and the rank of $A^{-1}$ is $n - 0$.

Find bases for each of the subspaces by:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
2 & -1 & 0 & 0 & 0 \\
9 & 5 & 2 & 1 & 0 \\
-2 & -2 & 1 & 0 & 1 \\
\end{bmatrix} = \mathbf{0}
\]

Assuming that all bases are found, two cofactor forms:

\[
\begin{bmatrix}
-6 & -6 & -6 & -6 & -6 \\
-6 & -6 & -6 & -6 & -6 \\
2 & 2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

Where $u_1$, $u_2$, $v_1$, $v_2$ are the eigenvectors of the matrices $A$ and $A^T$ respectively.

13. Show that any vector in $\mathbb{R}^n$ is a linear combination of $n$ linearly independent vectors if $n$ is greater than or equal to $\dim \mathbb{R}^n$.

14. Find a basis for the subspace of $\mathbb{R}^n$ spanned by the given vectors.